Further Pure 3 Past Paper Questions Pack A

Series and Limits

Pure 3 June 2001

Q	Solution	Marks	Total	Comments
4 (a)	$f'(x)=3\cos\left(3x+\frac{\pi}{4}\right)$	M1		Differentiate twice or use of $\sin(A+B)=\sin A\cos B+\cos A\sin B$
	$f''(x)=-9\sin\left(3x+\frac{\pi}{4}\right)$	A1		
	$f(x)=f(0)+f'(0)x+f''(0)x^2$			
	$=\frac{\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}x-\frac{9}{2}\frac{\sqrt{2}}{2}x^2$	B1	3	AG
(b)	$g(x)=\frac{\sqrt{2}}{2}\left(1+x+\frac{x^2}{2}\right)\left(1+3x-\frac{9}{2}x^2\right)$	M1		
	$=\frac{\sqrt{2}}{2}(1+4x$	A1		
	$-x^2)$	A1	3	
	Total		**6**	

Pure 3 June 2002

6(a)(i)	$f(x)=\ln(1-2x)$			
	$f'(x)=-2(1-2x)^{-1}$	M1A1		M1 – attempt to differentiate to get $\frac{*}{1-2x}$
	$f''(x)=-4(1-2x)^{-2}$	A1ft		ft on f′ $a(1-2x)^{-1}$
	$f'''(x)=-16(1-2x)^{-3}$	A1ft	4	ft on f″ $b(1-2x)^{-n}$
(ii)	$f(0)=0,\ f'(0)=-2,\ f''(0)=-4,\ f'''(0)=-16$	M1 M1		Use Maclaurin series Evaluate derivatives at x=0
	$\ln(1-2x)=0-2x-\frac{4x^2}{2}-\frac{16x^3}{6}$	A1	3	AG convincingly obtained, derivation correct Note hence – so use of formula for ln $(1+x)$ is not allowed
(b)	$2xe^x=2x\left(1+x+\frac{x^2}{2}+......\right)$			
	$=2x+2x^2+x^3$	B1		Allow expansion of xe^x from first principles.
	$2xe^x+\ln(1-2x)=x^3-\frac{8x^3}{3}$	M1		M1 – add expanded forms.
	$k=\frac{-5}{3}$	A1	3	Allow – 1.67
	Total		**10**	

Pure 3 January 2003

5 (a)(i)	$f(x) = \cos 2x$			
	$f'(x) = -2\sin 2x$	M1		$\cos 2x,\ k\sin 2x,\ l\cos 2x$
	$f''(x) = -4\cos 2x$	A1	2	
(ii)	$f(0) = 1 \quad f'(0) = 0 \quad f''(0) = -4$	B1ft		
	$\cos 2x \approx 1 + \frac{(-4)x^2}{2}$	M1		Use of $f(0)\ f'(0)\ f''(0)$
	$\approx 1 - 2x^2$	A1	3	Convincingly obtained
				Special Case $\cos x = 1 - \frac{x^2}{2}$ $\Rightarrow \cos 2x = 1 - \frac{(2x)^2}{2}$ $= 1 - 2x^2$ B1
(b)	$e^{-2x} + \sin x$			
	$= 1 - 2x + \frac{(-2x)^2}{2} + x$	M1 A1		Use given e^x approximation. i.e. use $\pm 2x$ in given expression for e^x, or attempt series for e^{-2x} from first principles.
	$= 1 - x + 2x^2$	A1	3	
(c)	$1 - x + 2x^2 = 1 - 2x^2$	M1		
	$x = \frac{1}{4}$	A1	2	Accept 0.25
				Special Case Correct solution to a soluble quadratic M1 A1
	Total		**10**	

Pure 3 June 2003

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathrm{f}(x)=(3x-2)^{-1}$	M1		
	$\mathrm{f}'(x)=-1\times 3(3x-2)^{-2}$	M1A1		M1 A0 for $-(3x-2)^{-2}$ or $3(3x-2)^{-2}$
	$\mathrm{f}''(x)=18(3x-2)^{-3}$	A1F	4	ft on $\mathrm{f}'(x)=k(3x-2)^{-2}$
(ii)	$\mathrm{f}(x)=\mathrm{f}(0)+\mathrm{f}'(0)x+\mathrm{f}''(0)\dfrac{x^2}{2}$	M1		Use $x=0$ in Maclaurin series (**NB** - need to see attempt at subsequent use)
	$\mathrm{f}(0)=-2^{-1}$, $\mathrm{f}'(0)=-3(-2)^{-2}$, $\mathrm{f}''(0)=18(-2)^{-3}$			
	$\mathrm{f}''(x)\approx-\frac{1}{2}-\frac{3}{4}x-\frac{9}{8}x^2$	A2, 1F	3	ft on (i); -1 EE
(b)	$(x-1)(x-2)=Ax(3x-2)+B(3x-2)+Cx^2$	M1		Any equivalent method M1 (as is)
	$x=0,\ B=-1,\ x=\frac{2}{3},\ C=1$	M1A1		Alternative: $x^2-3x+2=(3A+C)x^2+(-2A+3B)x-2B$ M1A1
	$x=1,\ 0=A+B+C$	m1		$3A+C=1$, $-2A+3B=-3$, $-2B=2$ } m1; $B=-1$, $A=0$, $C=1$ } A1
	but $B+C=0 \quad \Rightarrow A=0$	A1	5	$A=0$ convincingly shown SC(i): If A=0, then $B=-1$ and $C=1$ 3 marks Now $\frac{-1}{x^2}+\frac{1}{3x-2}=\frac{(x-1)(x-2)}{3x-2}$ so $A=0$ 2 marks SC(ii): candidate introduces extra x's e.g. $Ax^2(3x-2)+B(3x-2)x+Cx^2$ 1 mark standard method for A, B, C 1 mark (so max 2/5)
(c)	$\dfrac{(x-1)(x-2)}{(3x-2)}=x^2\left(-\frac{1}{x^2}-\frac{1}{2}-\frac{3}{4}x-\frac{9}{8}x^2\right)$	M1		SC: $(x-1)(x-2)\left(\frac{1}{2}-\frac{3}{4}x-\frac{9}{8}x^2\right)$ $=1-\frac{1}{2}x^2-\frac{3}{4}x^3+\frac{27}{8}x^3-\frac{9}{8}x^4$ Bl
	$=-1-\frac{1}{2}x^2-\frac{3}{4}x^3-\frac{9}{8}x^4$	A1	2	AG convincingly obtained
	Total		**14**	

Pure 3 January 2004

Q	Solution	Marks	Total	Comments
5 (a)(i)	$f(x) = e^{-2x}$ $\quad f'(x) = -2e^{-2x}$	B1		
	$f''(x) = 4e^{-2x}$	B1√	2	
(ii)	$f(x) = f(0) + f'(0)x + f''(0)\,\frac{x^2}{2}$			
	$f(0) = 1 \quad f'(0) = -2$	M1		Use $x = 0$ in Maclaurin series
	$f''(0) = 4$			
	$f(x) \approx 1 - 2x + 2x^2$	A1	2	AG convincingly obtained
(b)(i)	$\cos 3x \approx 1 - \frac{(3x)^2}{2}$	B1	1	
(ii)	$1 - 2x + 2x^2 = 1 - \frac{9}{2}x^2$	M1		Set up equation
	$\frac{13}{2}x^2 - 2x = 0$	m1		Rearrange to soluble form
	$x = \frac{4}{13}, \quad 0.308$	A1	3	Accept 0.31 Ignore $x = 0$
	Total		**8**	

Pure 3 June 2004

Q	Solution	Marks	Total	Comments
6(a)(i)	$f(x)=\sin\left(2x+\frac{\pi}{6}\right)$ $f'(x)=2\cos\left(2x+\frac{\pi}{6}\right)$ $f''(x)=-4\sin\left(2x+\frac{\pi}{6}\right)$	M1A1 A1	3	M1 for $\cos\left(2x+\frac{\pi}{6}\right)$ **Alternative** $f(x)=\sin\left(2x+\frac{\pi}{6}\right)$ $=\sin 2x\cos\frac{\pi}{6}+\cos 2x\sin\frac{\pi}{6}$ $=\frac{\sqrt{3}}{2}\sin 2x+\frac{1}{2}\cos 2x$ $f'(x)=\sqrt{3}\cos 2x-\sin 2x$ M1A1 $f''(x)=2\sqrt{3}\sin 2x-2\cos 2x$ A1 ($\cos\frac{\pi}{6}$ & $\sin\frac{\pi}{6}$ terms need not be simplified) If $f(x)=\sin\left(2x+\frac{\pi}{6}\right)$ expanded incorrectly i.e. $=\sin 2x+\sin\frac{\pi}{6}$ $f'(x)=2\cos 2x$ $f''(x)=-4\sin 2x$, must be fully correct for M1A0A0 If $f'(x)=\cos\left(2x+\frac{\pi}{6}\right)$ M1A0 $f''(x)=-2\sin\left(2x+\frac{\pi}{6}\right)$ A1F $x=0$, $f(0)=\frac{1}{2}$, $f'(0)=\frac{\sqrt{3}}{2}$, $f'(0)=-\frac{1}{2}$ M1A0 for part (ii)
(ii)	$f(x)=f(0)+f'(0)x+f''(0)\frac{x^2}{2}$ $f(x)=\frac{1}{2}+2\frac{\sqrt{3}}{2}x-4\frac{1}{2}\frac{x^2}{2}$ $f(x)\approx\frac{1}{2}+\sqrt{3}x-x^2$	M1 A1	2	Use $x=0$ in Maclaurin series, P.I. AG convincingly obtained: show how x^2 term is obtained
(b)	$\left(1-\left(1-\frac{x^2}{2}\right)\right)$ $\left(\frac{1}{2}+\sqrt{3}x-x^2\right)\frac{x^2}{2}\approx\frac{1}{4}x^2$ $k=\frac{1}{4}$	B1 M1A1	3	Use of $\cos x=1-\frac{x^2}{2}$; may be derived from first principles Either $k=\frac{1}{4}$ explicity stated or expression in question written with k replaced by $\frac{1}{4}$
	Total		**8**	

Pure 3 January 2005

Q	Solution	Marks	Total	Comments
4(a)(i)	$f(x)=e^{-3x}$ $\quad f'(x)=-3e^{-3x}$ $f''(x)=9e^{-3x}$	M1A1	2	
(ii)	$f(x)=f(0)+f'(0)x+f''(0)\frac{x^2}{2!}\ldots$	M1		
	$f(0)=1 \quad f'(0)=-3 \quad f''(0)=9$			
	$f(x)\approx 1-3x+\frac{9}{2}x^2$	A1	2	AG. Use of Maclaurin from (i) required.
(b)	$\ln(1+3x)\approx 3x-\frac{(3x)^2}{2}+\frac{(3x)^3}{3}$	M1		Allow $3x-\frac{3x^2}{2}+\frac{3x^3}{3}$ (or x^3)
	$=3x-\frac{9}{2}x^2+9x^3$	A1	2	CAO but allow $\frac{27}{3}x^3$
(c)	$3x-\frac{9}{2}x^2+9x^3-(2x-6x^2+9x^3)=0.1$	M1		
	$1.5x^2+x-0.1=0$	A1F		ft $\ln(1+3x)$ and simplification to $f(x)=0$. Correct quadratic any equivalent form
	$x=\frac{-1+\sqrt{1.6}}{3}=0.088$	M1A1	4	
	Total		**10**	

Pure 3 June 2005

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathrm{f}(x) = (2-3x)^{-1}$	M1		
	$\mathrm{f}'(x) = 3(2-3x)^{-2}$	M1A1		$-3(2-3x)^{-2}$ gets M1A0
	$\mathrm{f}''(x) = 18(2-3x)^{-3}$	A1F	4	ft only on $\mathrm{f}'(x) = -3(2-3x)^{-2}$ **SC** Attempt to use quotient rule M1 $\mathrm{f}'(x) = \frac{3}{(2-3x)^2}$ A1A1 $\mathrm{f}''(x) = \frac{6x(\pm 3)(2-3x)}{(2-3x)^4}$ A1F ft only on earlier sign error
(ii)	$\mathrm{f}(0) = \frac{1}{2}$ $\mathrm{f}'(0) = \frac{3}{4}$ $\mathrm{f}''(0) = \frac{18}{8}$	M1		use $x = 0$ in their derivatives
	$\mathrm{f}(x) \approx \frac{1}{2} + \frac{3}{4}x + \frac{1}{2}\times\frac{9}{4}x^2$	A1	2	AG
(b)(i)	$\cos 2x = 1 - \frac{(2x)^2}{2}$	B1	1	or from first principles brackets possibly implied further down
(ii)	$\frac{1}{2} + \frac{3}{4}x + \frac{9}{8}x^2 = 1 - 2x^2 - 2x$	M1		Maclaurin series $=$ cos series $-2x$ condone missing $-2x$
		m1		attempt to manipulate line above to form $g(x) = 0$
	$25x^2 + 22x - 4 = 0$	A1		
	$x = 0.15(46\ldots)$	A1	4	ignore other answer **SC** if simplified quadratic omitted: $x = 0.15$ 2/4 $x = 0.154(6)$ 4/4
	Total		**11**	

Pure 5 June 2001

Q	Solution	Marks	Total	Comments
1 (a)	$\frac{a^3x^3}{3!}$ … $\frac{a^5x^5}{5!}$	B1	1	do not accept $(ax)^3$ unless multiplied out later
(b)	Substitution $a = 3, 2$	M1		
	Limit $= \lim_{x\to 0} \frac{3\left(x - \frac{x^3}{6}\right) - \left(3x - \frac{27x^3}{6}\right) + O(x^5)}{2\left(x - \frac{x^3}{6}\right) - \left(2x - \frac{8x^3}{6}\right) + O(x^5)}$	A1√ A1√		for numerator for denominator
	$= 4$	M1 A1√	5	for cancelling terms and $\div x^3$ for accurate work and the answer
	Total		6	

Pure 5 June 2001

5 (a)	$\left(\frac{1}{y}\right)^{-k}\ln\left(\frac{1}{y}\right) = -\frac{\ln y}{y^{-k}} = -y^k \ln y$	M1A1		
	$x \to \infty, \quad y \to 0$ and $\lim y^k \ln y = 0$	A1	3	
(b)	$\int_1^\infty \frac{\ln x}{x^2}\,dx = \left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^\infty$	M1A1		
	$=1$	A1	3	
	Total		**6**	

Pure 5 January 2002

Q	Solution	Marks	Total	Comments
1	$\int_1^\infty \left(\frac{1}{x^2} - \frac{1}{1+x^2}\right)dx = \left[-\frac{1}{x} - \tan^{-1} x\right]_1^\infty$	B1		
	$= 1 - \frac{\pi}{4}$	B2,1,0	3	Must be of the correct form
	Total		**3**	

Pure 5 January 2002

4 (a)	$\sin^3 x = \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)^3$			
	$= x^3 + 3x^2\left(-\frac{x^3}{6} + \frac{x^5}{120}\right) + 3x\left(-\frac{x^3}{6} + \frac{x^5}{120}\right)^2$	M1 A2,1,0		Any form Also allow the M1A2 for $\sin^2 x = x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6$ o.e.
	$= x^3 - \frac{1}{2}x^5$	A1		Clearly shown
	$+\frac{13x^7}{120}$	A1	5	AG
(b)	$\sin x^3 = x^3 - \frac{x^9}{6}$	B1		
	$\lim_{x\to 0} \frac{\sin^3 x - \sin x^3}{x^5} = \frac{-\frac{1}{2}x^5 + \frac{13x^7}{120}}{x^5}$	M1		Ignore other terms here
	$= -\frac{1}{2}$	A1F	3	
	Total		**8**	

Pure 5 June 2002

Q		Solution	Marks	Total	Comments
4	(a)	$x = \cos\theta \quad \mathrm{d}x = -\sin\theta\,\mathrm{d}\theta$	M1A1		
		$\mathrm{I} = \int -\frac{\mathrm{d}x}{\sqrt{x}}$	A1√		√ sign error
		$= -2\sqrt{x} + c = -2\sqrt{\cos\theta}(+c)$	A1√	4	
	(b) (i)	Clear explanation	B1	1	Essentially for 'denominator is zero when $\theta = \frac{1}{2}\pi$' stated
	(ii)	$\int_0^{\pi/2} \frac{\sin\theta}{\sqrt{\cos\theta}}\,\mathrm{d}\theta = \left[-2\sqrt{\cos\theta}\right]_0^{\pi/2} = 2$	M1A1√	2	
		Total		**7**	

Pure 5 June 2002

7	(a)	$\cos x - 1 = 1 - \frac{x^2}{2} + \frac{x^4}{24} - 1 = -\frac{x^2}{2} + \frac{x^4}{24}$	B1	1	To earn B1, 2! and 4! must be expanded somewhere in the solution.
	(b)	$\mathrm{e}^{\cos x - 1} = \mathrm{e}^{-\frac{x^2}{2}}\mathrm{e}^{\frac{x^4}{24}}$ or $1 + \left(-\frac{x^2}{2} + \frac{x^4}{24}\right) + \frac{1}{2}\left(-\frac{x^2}{2} + \frac{x^4}{24}\right)^2$	M1		For second method there must be two terms in each bracket for M1 i.e. use of $-\frac{x^2}{2}$ alone is not acceptable.
		$= \left(1 - \frac{x^2}{2} + \frac{x^4}{8} \ldots\right)\left(1 + \frac{x^4}{24} \ldots\right)$ or substituted correctly as above	A1		
		$1 - \frac{x^2}{2}, + \frac{x^4}{6}$	A1A1	4	AG
	(c)	$\sin^2 x \approx x^2$	M1A1		[ignore higher powers of x]
		$\therefore \lim_{x \to 0} \frac{1 - \mathrm{e}^{\cos x - 1}}{\sin^2 x} = \frac{1}{2}$	A1√	3	
		Total		**8**	

Pure 5 January 2003

Q		Solution	Marks	Total	Comments
1	(a)	$\lim_{x\to\infty} x^k e^{-4x} = 0$ for $k > 0$	B1	1	Withhold if evidence of incorrect reasoning
	(b)	$\int_0^\infty xe^{-4x}dx = \left[\frac{xe^{-4x}}{-4}\right]_0^\infty - \int_0^\infty \frac{e^{-4x}}{-4}dx$	M1A1A1		
		$= 0$	A1		i.e. use of limit PI
		$+\left[-\frac{e^{-4x}}{16}\right]_0^\infty$	A1F		
		$= \frac{1}{16}$	A1	6	AG
	(c)	$\int_0^\infty x^2e^{-4x}dx = \left[\frac{x^2e^{-4x}}{-4}\right]_0^\infty - \int_0^\infty \frac{2xe^{-4x}}{-4}dx$	M1A1		
		$= \frac{1}{32}$	A1F	3	
		Total		**10**	

Pure 5 January 2003

Q		Solution	Marks	Total	Comments
3	(a)	Explanation	E1	1	
	(b)	$\int_{-2}^{2} \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$	M1A1		
		$= \sin^{-1}1 - \sin^{-1}(-1) = \pi$	A1F	3	ft $\sin^{-1}\frac{x}{4}$
		Total		**4**	

Pure 5 June 2003

2(a)(i)	$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24}$	B1	1	must show 2 and 24
(ii)	$\ln(1+x)$ used	M1		
	$\ln\left(1+\left(-\frac{x^2}{2}+\frac{x^4}{24}\right)\right)$			
	$= \left(-\frac{x^2}{2}+\frac{x^4}{24}\right) - \frac{1}{2}\left(-\frac{x^2}{2}+\frac{x^4}{24}\right)^2$	M1		if M0 allow B1 for $-\frac{x^2}{2}$
	$= -\frac{x^2}{2}$	A1		
	$-\frac{x^4}{12}$	A1	4	
(b)(i)	$\ln\sec x = -\ln\cos x$			
	$= \frac{x^2}{2} + \frac{x^4}{12}$	B1	1	
(ii)	$\lim_{x\to 0} \frac{\ln\sec x}{x^2} = \lim_{x\to 0} \frac{\frac{x^2}{2}+\frac{x^4}{12}}{x^2}$			if $-\frac{2}{x^2} - \frac{12}{x^4}$ used give zero
	$= \lim_{x\to 0} \frac{1}{2} + \frac{x^2}{12}$	M1		
	$= \frac{1}{2}$	A1F	2	
	Total		**8**	

Pure 5 June 2003

Q	Solution	Marks	Total	Comments
3(a)	Put $y = 1+x \quad dy = dx$	M1		Alternative by parts:
	$I = \int \frac{y-1}{y^2}dy$	A1		$-\frac{x}{1+x} + \int \frac{1}{1+x}dx$ M1A1A1
	$= \int\left(\frac{1}{y} - \frac{1}{y^2}\right)dy$	m1		$-\frac{x}{1+x} + \ln(1+x)$ A1 substitution of limits A2, 1, 0
	$= \left[\ln y + \frac{1}{y}\right]$	A1F		
	$= \left[\ln(1+x) + \frac{1}{1+x}\right]_0^k$			
	$= \ln(1+k) + \frac{1}{1+k} - 1$	A1F		
	$= \ln(1+k) - \frac{k}{1+k}$	A1	6	AG
(b)	As $k \to \infty$, $\ln(1+k) \to \infty$	B1		
	but $\frac{k}{1+k} \to 1$	B1		
	$\therefore$ does not exist	B1	3	dep on at least one of the previous two B marks earned
	Total		**9**	

Pure 5 January 2004

2 **(a)**	$u = 1 - x^2$ $\quad du = -2x dx$ or $x = \sin\theta$ $\quad dx = \cos\theta\, d\theta$	M1		
	$I = \int \frac{-du}{2u^{\frac{1}{2}}}$ or $I = \int \sin\theta\, d\theta$	A1		Limits not needed here
	$= \left[-u^{\frac{1}{2}}\right]$ or $[-\cos\theta]$	A1F		Limits not needed
	$= 1 - (1 - a^2)^{\frac{1}{2}}$	A1F	4	ft provided $p\left(1 - (1 - a^2)^{\frac{1}{2}}\right)$ where p is an integer
(b)	When $a = 1$, denominator is zero	E1	1	
(c)	$a = 1, I = 1$	M1A1F	2	
	Total		**7**	

Pure 5 January 2004

Q	Solution	Marks	Total	Comments
4 (a)(i)	$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24}$	B1		Simplification of factorials continued sensibly
	$\frac{1}{\cos x} = \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)^{-1}$	M1		
	$= 1 + \left(\frac{x^2}{2} - \frac{x^4}{24}\right) + \left(\frac{x^2}{2} - \frac{x^4}{24}\right)^2$	M1		
	$= 1 + \frac{x^2}{2}$	A1		AG
	$+ \frac{5x^4}{24}$	A1	5	AG
(ii)	$\tan x = \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)\left(1 + \frac{x^2}{2} + \frac{5x^4}{24}\right)$	M1A1		
	$= x + \frac{x^3}{3}$	A1F		Incorrect sin series
	$+ \frac{2x^5}{15}$ or $\frac{16x^5}{120}$	A1F	4	
(b)	$\lim\left(\frac{\tan 2x - 2x}{\tan x - x}\right) = \frac{2x + \frac{8x^3}{3} + \frac{64x^5}{15} - 2x}{x + \frac{x^3}{3} + \frac{2x^5}{15} - x}$	M1A1F		
	$= \frac{\frac{8}{3} + O(x^2)}{\frac{1}{3} + O(x^2)}$	A1F		
	$= 8$	A1F	4	
	Total		**13**	

Pure 5 June 2004

Q	Solution	Marks	Total	Comments
1(a)	$\frac{4}{x(x+4)}=\frac{1}{x}-\frac{1}{x+4}$	M1A1		Whole Q depends on the PFs
	$\mathrm{I}=\ln x-\ln(x+4)(+c)$	A1F	3	ft incorrect PFs
(b)(i)	$\mathrm{I}=[\ln x-\ln(x+4)]_0^1$	B1		attempt to put in limits
	$\ln x\to-\infty$ as $x\to 0\ \therefore$ no finite limit	E1	2	
(ii)	$\frac{x}{x+4}\to 1$ as $x\to\infty$	E1		a clear explanation is required
	$\therefore\ \mathrm{I}=\ln 1-\ln\frac{1}{5}$	M1		substitution of limits
	$=\ln 5$	A1F	3	O.E; no ln 1 in answer
	Total		**8**	

Pure 5 June 2004

2	$\cos^k x=\left(1-\frac{x^2}{2}...\right)^k$	M1		
	$=1-\frac{kx^2}{2}...$	A1		ignore higher powers of x
	$\lim_{x\to 0}\frac{1-\left(1-\frac{kx^2}{2}\right)}{x^2}=4$	M1		award only if some function of k appears
	$k=8$	A1F	4	
	Total		**4**	

Pure 5 January 2005

Q	Solution	Marks	Total	Comments
1(a)	$\sin 2x=2x-\frac{8x^3}{6}$	B1	1	Ignore extra terms
(b)	Use of $\left(1-\frac{x^2}{2}\right)$ and $\left(2x-\frac{8x^3}{6}\right)$	M1		
	$L=\lim_{x\to 0}\frac{2x\left(1-\frac{x^2}{2}\right)-\left(2x-\frac{8x^3}{6}\right)+0\left(x^5\right)}{x^3}$	A1F		Condone $0\left(x^5\right)$ missing
	$=\lim_{x\to 0}\frac{-x^3+\frac{8x^3}{6}+0\left(x^5\right)}{x^3}$	A1F		
	$=\frac{1}{3}$	A1F	4	
	Total		**5**	

Pure 5 January 2005

3(a)	$\mathrm{I} = \left[2\sqrt{x}\ln x\right]_k^1 - \int_k^1 2\sqrt{x}\frac{1}{x}\mathrm{d}x$	M1A1 A1		
	$\left[-4\sqrt{x}\right]_k^1$	A1F		
	$= 4\left(\sqrt{k}-1\right) - 2\sqrt{k}\ln k$	A1	5	AG
(b)	Exists since $\sqrt{k}\ln k \to 0$ as $k \to 0$	E1		Clear explanation
	value is – 4	A1F	2	If M0 earlier, allow B1 for – 4
	Total		**7**	

Pure 5 June 2005

3	$\ln(1+x) \simeq x - \frac{x^2}{2}$			
	and $\cos x \simeq 1 - \frac{x^2}{2}$ both used	M1		
	$\lim\limits_{x\to 0}\frac{x\ln(1+x)}{1-\cos x} \simeq \lim\limits_{x\to 0}\frac{x^2 + 0\left(x^3\right)}{\frac{x^2}{2} + 0\left(x^4\right)}$	A1		ignore errors in powers of $x > 2$
	Dividing by x^2, ie $\lim\limits_{x\to 0}\frac{1+0(x)}{\frac{1}{2} + 0\left(x^2\right)}$	m1		PI
	$= 2$	A1F	4	**Alternative method:** L'Hôpital's rule used twice M1 Correct differentiation A1 Putting $x = 0$ m1 (allow this m1 if $x = 0$ gives a finite limit) Result 2 A1F
	Total		**4**	

Pure 5 June 2005

5(a)	$u = \ln x \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$	B1		
	$\mathrm{I} = \int \frac{\mathrm{d}u}{u} = \ln u = \ln(\ln x) + c$	M1A1	3	M1 for $\ln u$ A1 for $\ln(\ln x)$ condone omission of c
(b)(i)	Clear reason why improper	E1	1	
(ii)	When $x = 1$, $\ln(\ln 1) = \ln 0$ and does not exist	E2	2	E2 for clear reasoning, E1 if vague
	Total		**6**	

Polar Coordinates

Pure 5 June 2001

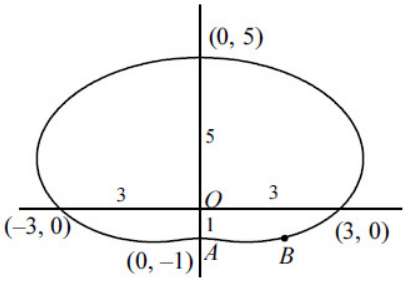
Q	Solution	Marks	Total	Comments
1 (a)	$\sin ax = ax - \frac{a^3x^3}{3!} + \frac{a^5x^5}{5!}$	B1	1	do not accept $(ax)^3$ unless multiplied out later
(b)	Substitution $a = 3, 2$	M1		
	Limit $= \lim_{x\to 0} \frac{3\left(x - \frac{x^3}{6}\right) - \left(3x - \frac{27x^3}{6}\right) + 0\left(x^5\right)}{2\left(x - \frac{x^3}{6}\right) - \left(2x - \frac{8x^3}{6}\right) + 0\left(x^5\right)}$	A1√ A1√		for numerator for denominator
	$= 4$	M1 A1√	5	for cancelling stems and $\div x^3$ for accurate work and the answer
	Total		**6**	

Pure 5 June 2001

Q	Solution	Marks	Total	Comments
4 (a)(i)	$y = 2(1+\cos\theta)\sin\theta \to \frac{dy}{d\theta} = 2\cos\theta + 2\cos 2\theta$	M1A1		
	$\cos\theta + 2\cos^2\theta - 1 = 0$	M1A1		
	$(2\cos\theta - 1)(\cos\theta + 1) = 0 \quad \therefore \cos\theta = \frac{1}{2}, \theta = \frac{\pi}{3}$	A1	5	
(ii)	P is $\left(3, \frac{\pi}{3}\right)$; Q is $\left(3, -\frac{\pi}{3}\right)$	B1B1	2	
(iii)	$r\cos\theta = 1.5$	M1 A1	2	for OA
(b)(i)	Area of $\Delta OPQ = \frac{1}{2} \times 9 \sin\frac{2\pi}{3}$	M1		
	$= \frac{9\sqrt{3}}{4}$	A1	2	AG
(ii)	Area A = area of R + area of ΔOPQ	M1		
	$= 2 \times \frac{1}{2} \int_0^{\frac{\pi}{3}} 4(1+\cos\theta)^2 \, d\theta$	A1		
	Use of $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	M1		
	Area A $= 4\int_0^{\frac{\pi}{3}} \left[(1+2\cos\theta) + \frac{1}{2}(1+\cos 2\theta)\right] d\theta$	A1√		
	$= \left[6\theta + 8\sin\theta + \sin 2\theta\right]_0^{\frac{\pi}{3}}$	A1√		
	$= 2\pi + \frac{9\sqrt{3}}{2}$	A1√		
	Area R $= 2\pi + \frac{9\sqrt{3}}{4}$	A1	7	AG
	Total		**18**	

Pure 5 January 2002

Q	Solution	Marks	Total	Comments
5 (a)(i)	r values 1, 2	B1		
	Points shown $\left(1,-\frac{\pi}{2}\right)$ $\left(2,-\frac{\pi}{6}\right)$	B1F	2	or $(0,-1)$ $(2\cos\frac{\pi}{6}, -1)$ Allow approximate positions
(ii)	Use of geometry to find y values	M1A1	2	
(iii)	Symmetrical about $\theta = \frac{\pi}{2}$	B1		
	Strategic points to be labelled or tabulated Dent at A	B1 B1	3	



Q	Solution	Marks	Total	Comments
(b)	$A = \frac{1}{2}\int_0^{2\pi}(3+2\sin\theta)^2\,d\theta$	M1		
	$= \frac{1}{2}\int_0^{2\pi}(9+12\sin\theta+4\sin^2\theta)\,d\theta$	A1		
	Use of $\cos 2\theta = 1-2\sin^2\theta$	m1		If m0, allow B1 for integrating $9+12\sin\theta$ correctly
	$A = \frac{1}{2}\int_0^{2\pi}(11+12\sin\theta-2\cos 2\theta)\,d\theta$	A1F		
	$= \frac{1}{2}[11\theta-12\cos\theta-\sin 2\theta]_0^{2\pi}$	A1F		
	$= 11\pi$	A1F	6	For incorrect limits penalise here
(c)	OP, OQ, OM, ON : any two Other two; simplified	M1A1 A1A1		
	$= 9-4\sin^2\theta+9-4\cos^2\theta$	A1		
	$= 14$	A1	6	
	Total		**19**	

Pure 5 June 2002

Q	Solution	Marks	Total	Comments
1	$A = \frac{1}{2}\int_0^{2\pi} a^2\sin^2\frac{1}{2}\theta\ d\theta$	M1A1B1		M1 for $\int\frac{1}{2}r^2 d\theta$ used A1 if used correctly B1 for limits M0 if $\cos 2\theta$ used
	$= \frac{1}{2}\int_0^{2\pi} a^2\left(\frac{1-\cos\theta}{2}\right)d\theta$	M1		
	$= \left[\frac{1}{2}a^2\left(\frac{\theta}{2}-\frac{\sin\theta}{2}\right)\right]_0^{2\pi}$	A1		CAO
	$= \frac{1}{2}\pi a^2$	A1√	6	
	Total		**6**	

Pure 5 June 2002

3 (a)	$\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$	B1	1	
(b) (i)	$r = 2\frac{x}{r} - 4\frac{y}{r}$	M1		
	use of $x^2 + y^2 = r^2$	M1		
	$x^2 + y^2 = 2x - 4y$	A1	3	
(ii)	$(x-1)^2 + (y+2)^2 = 5$	M1A1√		
	Centre $(1, -2)$, radius $\sqrt{5}$	A1√	3	
	Total		**7**	

Pure 5 January 2003

2 (a)	Sketch: Correct starting and finishing points	B1		
	Going the right way round the origin	B1		
	Steadily decreasing in distance from the origin	B1	3	
(b)(i)	$A = \frac{1}{2}\int_{\frac{\pi}{4}}^{\alpha} \frac{1}{\theta^2}\,\mathrm{d}\theta$	M1A1		
	$= \left[-\frac{1}{2\theta}\right]_{\frac{\pi}{4}}^{\alpha}$	A1F		Provided of the form $\frac{k}{\theta}$
	$= \frac{2}{\pi} - \frac{1}{2\alpha}$	A1	4	AG
(ii)	$\frac{2}{\pi} - \frac{1}{2\alpha} = \frac{1}{2}\left(\frac{2}{\pi} - \frac{1}{4\pi}\right)$	M1		
	$\frac{1}{2\alpha} = \frac{2}{\pi} - \frac{1}{\pi} + \frac{1}{8\pi}$	A1		
	$\alpha = \frac{4\pi}{9}$	A1F	3	α must be in range $\frac{\pi}{4} - 2\pi$
	Total		**10**	

Pure 5 June 2003

Q	Solution	Marks	Total	Comments
6(a)(i)	$x^2+y^2=r^2=16 \quad \therefore r=4$	B1	1	needs some explanation
(ii)	$r\cos\theta \; r\sin\theta=4$	M1		
	$r^2\sin 2\theta=8$	A1		
	$r^2=8\operatorname{cosec}2\theta$	A1	3	AG
(b)	At points of intersection			
	$\dfrac{8}{\sin 2\theta}=16$ or $8\cos 2\theta=16$	M1		
	$\sin 2\theta=\dfrac{1}{2}$	A1		
	$2\theta=\dfrac{\pi}{6}, \quad \theta=\dfrac{\pi}{12}$	A1		withhold if calculator used
	Or $2\theta=\dfrac{5\pi}{6}, \quad \theta=\dfrac{5\pi}{12}$	B1	4	
(c)	Area of sector $AOB=\dfrac{1}{2}4^2\left(\dfrac{5}{12}\pi-\dfrac{1}{12}\pi\right)$			
	$=\dfrac{8\pi}{3}$	B1F		ft incorrect $\dfrac{5\pi}{12}$
	Area between $xy=4$ and lines OA, OB			
	$=\displaystyle\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}}\frac{1}{2}r^2 d\theta=\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}}4\operatorname{cosec}2\theta\,d\theta$	M1		treat missing $\dfrac{1}{2}$ as A-error ignore limits here
	$=-\dfrac{4}{2}\ln(\operatorname{cosec}2\theta+\cot 2\theta)$	A1		or $\dfrac{4}{2}\ln\tan\theta$
	$=-2\ln\left(\operatorname{cosec}\dfrac{5\pi}{6}+\cot\dfrac{5\pi}{6}\right)$			
	$+2\ln\left(\operatorname{cosec}\dfrac{\pi}{6}+\cot\dfrac{\pi}{6}\right)$	m1		For putting limits into their integral (not 0 and $\dfrac{\pi}{2}$ for limits)
	$=-2\ln(2-\sqrt{3})+2\ln(2+\sqrt{3})$	A1F		
	$=2\ln\left(\dfrac{2+\sqrt{3}}{2-\sqrt{3}}\right)$	A1F		
	$=4\ln(2+\sqrt{3})$	A1	7	
	Total		**15**	

Pure 5 January 2004

Q	Solution	Marks	Total	Comments
3 (a)	$A=\frac{1}{2}\int_{\theta_1}^{\theta_2} e^{2k\theta}d\theta$	M1A1		A1 for $e^{2k\theta}$
	$=\frac{1}{4k}\left[e^{2k\theta}\right]_{\theta_1}^{\theta_2}$	A1		
	$=\frac{1}{4k}\left[e^{2k\theta_2}-e^{2k\theta_1}\right]$			
	$=\frac{1}{4k}\left(r_2^2-r_1^2\right)$	A1	4	AG
(b)(i)	at $K, e^{\theta}=2$	M1		
	$\theta=\ln 2$			
	K is $(2, \ln 2)$	A1	2	Accept $(2, 0.69(3))$
(ii)	Area of sector of circle is $\frac{1}{2}\times 2^2\ln 2$			
	$=2\ln 2$	B1		
	Area under curve by (a) is $\frac{1}{4}\left(2^2-1^2\right)$	M1		
	$=\frac{3}{4}$	A1		
	Shaded area $=2\ln 2-\frac{3}{4}$	M1A1F	5	M0 if added or subtracted the wrong way round ft simple slips
	Total		**11**	

Pure 5 June 2004

Q	Solution	Marks	Total	Comments
4	$2=r+r\cos\theta$	M1		
	$=r+x$	B1		i.e. $x=r\cos\theta$ used relevantly
	$2-x=r$	A1		
	$(2-x)^2=x^2+y^2$	M1		For relevant use of $r=\sqrt{x^2+y^2}$
	$4-4x+x^2=x^2+y^2$	A1		
	$y^2=4(1-x)$	A1F	6	Or $y^2=4-4x$ o.e. ft simple arithmetical errors only
	Total		**6**	

Pure 5 June 2004

6(a)	$R_1+R_2 = \frac{1}{2}\int_{-(\pi-\alpha)}^{\alpha} 4(1-\cos\theta)^2\,\mathrm{d}\theta$	M1A1		M1 for use of formula A1 for correct limits (appearing at any point)
	$(1-\cos\theta)^2 = 1-2\cos\theta+\cos^2\theta$	A1		
	$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ used	M1		
	$\mathrm{I} = \left[3\theta - 4\sin\theta + \frac{\sin 2\theta}{2}\right]$	A1F		
	$a=3,\ b=-8$	A1A1	7	CAO
(b)	$OA = 2\,(1-\cos\alpha)$	B1		
	$OB = 2\,(1-\cos(-\pi+\alpha))$	B1		Could use $\pi+\alpha$
	$AB = 4$	B1	3	
	Total		**10**	

Pure 5 January 2005

Q	Solution	Marks	Total	Comments
5(a)	$\dfrac{2}{\cos\theta}=3+2\cos\theta$	M1		or corresponding results in r
	$2\cos^2\theta+3\cos\theta-2=0$	A1		
	$(2\cos\theta-1)(\cos\theta+2)=0$	m1		
	$\cos\theta=\dfrac{1}{2},\ \cos\theta\neq-2$	A1		
	at $A,\ \theta=\dfrac{\pi}{3}$	A1		
	A and B are $\left(4,\pm\dfrac{1}{3}\pi\right)$	A1	6	Accept $\left(4,\dfrac{5\pi}{3}\right)$
(b)	Area S bounded by C, OA, OB			
	$S=\dfrac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(3+2\cos\theta)^2\,d\theta$	M1		Ignore limits here
	$=\int_0^{\frac{\pi}{3}}(9+12\cos\theta+4\cos^2\theta)\,d\theta$	A1		
	Use of $\cos2\theta=2\cos^2\theta-1$	m1		For an attempt to express $\cos^2\theta$ in terms of $\cos2\theta$
	$S=\int_0^{\frac{\pi}{3}}(11+12\cos\theta+2\cos2\theta)\,d\theta$	A1F		
	$=\left[11\theta+12\sin\theta+\sin2\theta\right]_0^{\frac{\pi}{3}}$	A1F		
	$=\dfrac{11\pi}{3}+\dfrac{13\sqrt{3}}{2}$	A1F		Correct limits needed here
	Area of $\Delta\ OAB=\frac{1}{2}4^2\sqrt{\frac{3}{2}}=4\sqrt{3}$	M1A1		Allow M1 for $\left[2\tan\theta\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$
	Area of $R=\dfrac{11\pi}{3}+\dfrac{13\sqrt{3}}{2}-4\sqrt{3}$			
	$=\dfrac{11\pi}{3}+\dfrac{5\sqrt{3}}{2}$	A1F	9	
	Total		**15**	

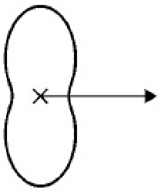
Pure 5 June 2005

4(a)(i)	$\sin 3\theta = 0$ when $\theta = 0,\ \frac{1}{3}\pi$	B1B1	2	
(ii)	Area of loop $= \frac{1}{2}\int_0^{\frac{1}{3}\pi} \sin^2 3\theta \, d\theta$	M1		ignore limits
	$= \frac{1}{2}\int_0^{\frac{1}{3}\pi} \frac{1}{2}(1 - \cos 6\theta) d\theta$	m1		
	$= \frac{1}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\frac{1}{3}\pi}$	A1		
	$= \frac{1}{12}\pi$	A1	4	AG
(b)	Area of $R = \frac{1}{3}\left(\pi \times 1^2 - 3 \times \frac{\pi}{12}\right)$	B1B1		B1 for each part, ie for two relevant areas to be subtracted
	$= \frac{1}{4}\pi$	B1	3	OE; must be correct
	Total		**9**	

Pure 5 June 2005

Q	Solution	Marks	Total	Comments
7(a)(i)	$4x^2 + 4y^2 = x^2 - 4x + 4$	M1		or any correct method
	$4(x^2 + y^2) = (2 - x)^2$	A1	2	AG
(ii)	Use of $x = r\cos\theta,\ x^2 + y^2 = r^2$	M1M1		
	$4r^2 = (2 - r\cos\theta)^2$	A1		
	$2r = \pm(2 - r\cos\theta)$	A1		condone omission of $\pm$ sign
	$\frac{2}{r} = 2 + \cos\theta$	m1A1	6	AG
(b)	If $\theta = \alpha$ at P, $\theta = \alpha + \pi$ at Q	M1A1		or $x - \pi$ at Q
	$\left.\begin{array}{l}\frac{2}{OP} = 2 + \cos\alpha \\ \frac{2}{OQ} = 2 + \cos(\alpha + \pi)\end{array}\right\}$	M1		
	$= 2 - \cos\alpha$	A1		
	$\frac{2}{OP} + \frac{2}{OQ} = 4$	M1		
	$\frac{1}{OP} + \frac{1}{OQ} = 2$	A1	6	AG
	Total		**14**	

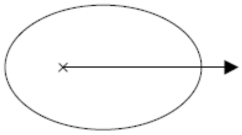
Pure 3(B) January 2002

Question Number and part	Solution	Marks	Total Marks	Comments
10(a)(i)	max = 6	B1		
	when $\theta = -\frac{\pi}{2}$	B1		or $-\frac{\pi}{2}$
	min = 2	B1		
	when $\theta = 0$	B1	4	or π
(ii)				
	graph 'correct' $0 < \theta < \frac{\pi}{2}$	B1		or any other quadrant condone cusps
	symmetry about $\theta = 0$ or $\theta = \frac{\pi}{2}$	B1		
	good attempt at graph	B1	3	
(b)(i)	$2 + 4(1 - \cos^2\theta) = 5\cos\theta$	M1		
	$4c^2 + 5c - 6 = 0$			
	$(4c - 3)(c + 2) = 0$	m1		factors or attempt to solve
	$\Rightarrow \cos\theta = \frac{3}{4}$	A1	3	
(ii)	$r = \frac{15}{4} = 3.75$	B1√		
	$\theta = \cos^{-1}(\frac{3}{4}) = 0.72$	B1		
	$\theta = -\cos^{-1}(\frac{3}{4})$	B1√	3	ft their $\cos\theta$ in either equation, coords or $[r, \theta]$ separately
	Total		**13**	

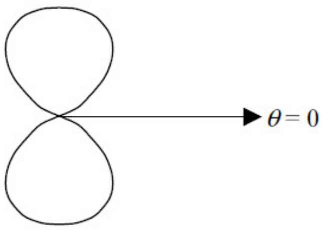
Pure 3(B) June 2002

Question Number and part	Solution	Marks	Total marks	Comments
6(a)(i)	Max $r = 4$ Min $r = 0$	B1 B1	 2	**or** $r \le 4$ **or** $r \ge 0$ sc Allow B1 for $0 < r < 4$
(ii)	$\theta = 0$ at max $\theta = \frac{\pi}{2}$ at min	B1 B1	 2	**or** π Accept $90°$; **or** $\theta = -\frac{\pi}{2}$ etc
(iii)		B1 B1 B1	 3	One quadrant correct Symmetry about $\theta = 0$ Symmetry about $\theta = \frac{\pi}{2}$
(b)(i)	Translation through $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	M1 A1	 2	Translated 1 in x-direction M1, A1 Moved/shifted one unit to right M1, A0 Moved one unit M0 Translated in y-direction M0
(ii)	$r^2 \cos^2 \theta - 2r\cos\theta + 1$ $+ r^2 \sin^2 \theta = 1$ Use of $\cos^2 \theta + \sin^2 \theta = 1$ $\Rightarrow r^2 = 2r\cos\theta \Rightarrow r = 2\cos\theta$	M1 A1 m1 A1	 4	**or** $x^2 - 2x + 1 + y^2 = 1$ $x^2 + y^2 = 2x$ $x^2 + y^2 = r^2, \quad x = r\cos\theta$ ag
(c)	$4\cos^2 \theta = 2\cos\theta$ $\cos\theta = 0$ or $\cos\theta = \frac{1}{2}$ Intersection at pole , or stating $r = 0$ Also at $\left[1, \frac{\pi}{3}\right]$ and $\left[1, \frac{-\pi}{3}\right]$	M1 B1 A1 B1 B1	 5	 **or** $\theta = \frac{\pi}{3}$ $\left[0, \frac{\pi}{2}\right]$ or $\left[0, -\frac{\pi}{2}\right]$ May state $r = 1$, $\theta = \pm\frac{\pi}{3}$ (but must have both points) B1 first correct point; B2 all three points
	Total		**18**	

Pure 3(B) January 2003

Q	Solution	Marks	Total	Comments
8(a) (i)	Max $r = 7$, $\theta = 0$ at max	B1, B1		
	Min $r = 3$, $\theta = \pi$ at min	B1, B1	4	
(ii)		B1 B1 B1	3	One quadrant correct Symmetry about $\theta = 0$ Good graph
(b)	$r\cos\theta = x$	M1		
	$x = 3$ is straight line	A1	2	
(c)(i)	$2\cos^2\theta + 5\cos\theta - 3 = 0$	M1		
	$(2\cos\theta - 1)(\cos\theta + 3) = 0$	m1		
	$\cos\theta \neq -3 \quad \Rightarrow \cos\theta = \frac{1}{2}$	A1	3	
(ii)	$r = 6$, $\theta = \dfrac{\pi}{3}$	B1, B1		
	$\theta = -\dfrac{\pi}{3}$	B1√	3	
	Total		**15**	

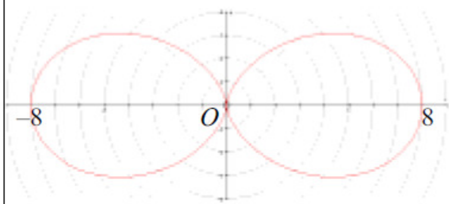
Pure 3(B) June 2003

Question Number and part	Solution	Marks	Total	Comments
6(a)(i)	$\sin(-\theta) = -\sin\theta$	M1		Must be general angle
	$f(-\theta) = 2(-\sin\theta)^2 = 2\sin^2\theta = f(\theta)$	A1	2	
(ii)	$\theta = \dfrac{\pi}{2}$	B1		accept $90°$ or 1.57(0796…..rads)
	$\theta = -\dfrac{\pi}{2}$	B1	2	**Or** -1.57(0796…..rads) Must be radians for second mark
(iii)	Range $0 \le f(\theta) \le 2$	B2	2	B1 only for either "end" correct or < used or θ used instead of f(θ)
(b)		B1 B1√ B1√	3	One quadrant correct Symmetry about $\theta = 0$ Symmetry about $\theta = \dfrac{\pi}{2}$
(c)(i)		M1 A1	2	"loop" generally above initial line single "circle" drawn fairly accurately
(ii)	$y = r\sin\theta$	B1		Seen or used
	$r^2 = 2r\sin\theta$	M1		**Or** attempt to eliminate both r and θ
	$x^2 + y^2 = 2y$	A1	3	oe such as $\sqrt{x^2 + y^2} = \dfrac{2y}{\sqrt{x^2 + y^2}}$ **sc** B3 for correct equation of circle with no working.
	Total		**14**	

Pure 3(B) January 2004

Question number and part	Solution	Marks	Total	Comments
8(a)	$r_{max} = 2$ when $\theta = \frac{1}{4}\pi$ and $\theta = -\frac{3}{4}\pi$	B1 B1 B1	 3	
(b)	$\sin 2\theta = -1 \Rightarrow 2\theta = \frac{3}{2}\pi, \ldots$	M1 A1		
	giving $\theta = \frac{3}{4}\pi$, $\theta = -\frac{1}{4}\pi$	A1 A1	4	Penalise degrees max. once; ignore correct out-of-range answers
(c)	Use of $r = \sqrt{x^2 + y^2}$	B1		
	Use of either $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$	M1		
	$\sqrt{x^2 + y^2} = 1 + \frac{2xy}{x^2 + y^2}$	A1	3	Any correct form at earliest stage
	Total		**10**	

Pure 3(B) January 2005

Question Number and Part	Solution	Marks	Total	Comments
8(a)(i)	Translation (// x-axis), vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$	M1 A1	2	B1 for equivalent correct description without "translation"
(ii)	$(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$ $r^2(\cos^2\theta + \sin^2\theta) - 4r\cos\theta + 4 = 4$ Use of $c^2 + s^2 = 1$ $(r \neq 0) \Rightarrow r = 4\cos\theta$	M1 A1 B1 A1	 4	Backwards approach is fine **ag**
(b)(i)	$r_{max} = 8$, $r_{min} = 0$	B1 B1	2	
(ii)	 	B1 B1 B1	 3	Symmetry in $\theta = \frac{1}{2}\pi$ Symmetry in $\theta = 0$ All correct
(c)	Equating $8\cos^2\theta = 4\cos\theta$ and solving $\theta = \frac{1}{3}\pi$ and $r = 2$ 2nd point $\theta = -\frac{1}{3}\pi$, $r = 2$	M1 A1 A1 A1√	 4	 Or ft $2\pi - (1^{st}\ \theta)$, same r
	Total		**15**	

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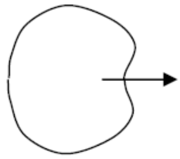
Q	Solution	Marks	Total	Comments
6(a)(i)	Maximum value of $r = 5$	B1		
	when $\theta = \pi$	B1		condone angles mod 2π
	Minimum value of $r = 1$	B1		
	when $\theta = 0$	B1	4	
(ii)	Symmetry about $\theta = 0$	B1		
	Correct graph – approx 5: 1 ratio	B1	2	
(b)(i)	$8c^2 + 2c - 3 = 0$			
	$\Rightarrow (2c-1)(4c+3) = 0$	M1		Attempt to factorise or solve quad eqn
	$\cos\theta = \frac{1}{2}, \quad \cos\theta = -\frac{3}{4}$	A1	2	
(ii)	Use of $r = 3 - 2\cos\theta$ to find r	M1		or using $r = 8\cos^2\theta$
	$\left[2, \frac{\pi}{3}\right], \left[2, -\frac{\pi}{3}\right], \left[\frac{9}{2}, \cos^{-1}(-0.75)\right]$	A1√		one pair of matching r and θ ft
		A1√		second pair of matching r and θ ft
	$\left[\frac{9}{2}, -\cos^{-1}(-0.75)\right]$	A1	4	All 4 points correct
	Total		**12**	