

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS

6993

Summer 2005

Monday

20 JUNE 2005

Morning

2 hours

Additional materials:

Answer booklet

Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Additional sheets of graph paper should be securely attached to your answer booklet.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given correct to three significant figures where appropriate.
- The total number of marks for this paper is 100.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

- 1 Use calculus to show that there is a maximum point at $x = 3$ on the curve $y = 9x^2 - 2x^3$ and find the coordinates of this point. [5]
- 2 The function $f(x)$ is defined by $f(x) = x^3 - 4x^2 + 5x - 2$.
- (i) Find the remainder when $f(x)$ is divided by $(x + 2)$. [2]
- (ii) Show that $(x - 1)$ is a factor of $f(x)$. [1]
- (iii) Hence solve the equation $f(x) = 0$. [4]
- 3 A triangle has sides 8 cm, 7 cm and 12 cm. Calculate the largest angle of the triangle, correct to the nearest degree. [5]
- 4 Find the values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying the equation
- $$4\sin \theta = 3\cos \theta.$$
- Give your answers to the nearest 0.1 degree. [4]
- 5 In a large batch of glasses, 14% are defective. From this batch 8 glasses are selected at random. Calculate which is more likely:
- (A) none of the glasses is defective,
- (B) at least two of the glasses are defective. [7]
- 6 (i) Expand $\left(x - \frac{1}{x}\right)^4$ using the binomial expansion. Show all your working. [4]
- (ii) Explain why the substitution $x = 1$ will help to justify your answer. [1]
- 7 The gradient function of a curve is given by $\frac{dy}{dx} = a + bx$. Find the values of a and b and the equation of the curve given that it passes through the points $(0, 2)$, $(1, 8)$ and $(-1, 2)$. [7]

- 8 A car moves in a straight line. Its velocity in metres per second, t seconds after passing a point A, is given by the equation

$$v = 27 - \frac{1}{8}t^3.$$

It comes to rest at a point B.

- (i) Show that the car is at B when $t = 6$. [1]

- (ii) Find the distance AB. [5]

- 9 (i) Using the identity $\cos^2 \theta + \sin^2 \theta = 1$, show that the equation

$$2\cos^2 \theta + \sin \theta = 2$$

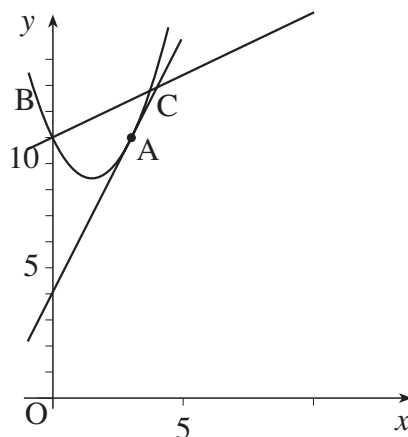
can be written as $2\sin^2 \theta - \sin \theta = 0$. [2]

- (ii) Hence find all values of θ in the range $0^\circ \leq \theta \leq 180^\circ$ satisfying the equation

$$2\cos^2 \theta + \sin \theta = 2. [4]$$

Section B

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The curve shown has equation $y = \frac{2}{3}x^2 - 2x + 10$.

- (i) Find the equation of the tangent to the curve at A (3, 10). [4]
- (ii) Show that the equation of the normal to the curve at B (0, 10) is $2y - x = 20$. [3]
- (iii) Find the coordinates of the point C where these two lines intersect. [3]
- (iv) Calculate the length BC. [2]

- 11 A small factory makes two types of components, X and Y. Each component of type X requires materials costing £18 and each component of type Y requires materials costing £11. In each week materials worth £200 are available.

Each component of type X takes 7 man hours to finish and each component of type Y takes 6 man hours to finish. There are 84 man hours available each week.

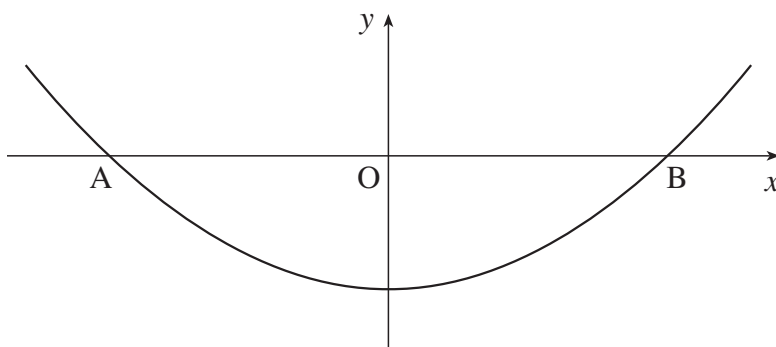
Components cannot be left part-finished at the end of the week. In addition, in order to satisfy customer demands, at least 2 of each type are to be made each week.

- (i) The factory produces x components of type X and y components of type Y each week. Write down four inequalities for x and y . [4]
- (ii) On a graph draw suitable lines and shade the region that the inequalities do not allow. (Take 1 cm = 1 component on each axis.) [5]
- (iii) If all components made are sold and the profit on each component of type X is £70 and on each component of type Y is £50, find from your graph the optimal number of each that should be made and the total profit per week. [3]

(Do not forget to hand in your graph paper with your answer booklet.)

- 12** (i) A circle has equation $x^2 + y^2 - 2x - 4y - 20 = 0$. Find the coordinates of its centre, C, and its radius. [3]
- (ii) Find the coordinates of the points, A and B, where the line $y = x + 2$ cuts the circle. [5]
- (iii) Find the angle ACB. [4]

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The shape of the bed of a river is to be modelled mathematically. The diagram represents a cross-section of the river. A and B on the x -axis represent points on opposite banks of the river at water level. (Units are metres.)

The shape of the river bed between A and B is modelled by the equation

$$y = \frac{3}{16}(x^2 - 16).$$

- (i) Find the coordinates of A and B and hence state the width of the river represented by the length AB. [2]
- (ii) Find the depth of the river at its deepest point. [2]
- (iii) Find the area of the cross-section of the river. [5]
- (iv) The river flows at 20 metres a minute. You should assume that this rate applies to all points of this cross-section.
- Find the volume of water that flows through this cross-section per minute. [1]
- (v) Give two reasons why this model may not be a good model. [2]

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