OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS 6993

Summer 2006

Thursday **15 JUNE 2006** Afternoon 2 hours

Additional materials:
16 page answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Additional sheets of graph paper should be securely attached to your answer booklet.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 100.

This question paper consists of 6 printed pages and 2 blank pages.

Section A

1 Find $\int_1^3 (x^2 + 3)\, dx$. [4]

2 Adam and Beth set out walking from a point P. After one hour Adam is 3.6 kilometres due north of P and Beth is 2.5 kilometres from P on a bearing of 035°.

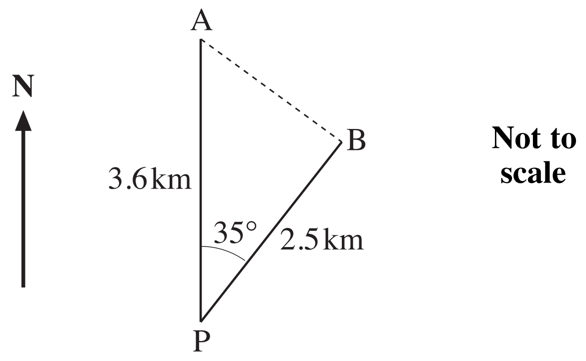

Fig. 2

Calculate how far they are apart at this time. Give your answer correct to 2 significant figures. [4]

3 Calculate the values of x in the range $0° < x < 360°$ for which $\sin x = 2\cos x$. [4]

4 **(i)** Find the distance between the points $(2, 3)$ and $(7, 9)$. [2]

(ii) Hence find the equation of the circle with centre $(2, 3)$ and passing through the point $(7, 9)$. [2]

5 Solve the inequality $x^2 + 4x > 5$. [5]

6 A curve has gradient given by $\frac{dy}{dx} = 2x + 2$. The curve passes through the point $(3, 0)$. Find the equation of the curve. [5]

7 **(i)** Show that the two lines whose equations are given below are parallel.

$$y = 4 - 2x$$
$$4x + 2y = 5$$

[2]

(ii) Find the equation of the line which is perpendicular to these two lines and which passes through the point $(1, 6)$. [3]

8 **(i)** By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$3x + 2y \leqslant 18$$
$$y \leqslant 3x$$
$$y \geqslant 0$$
[5]

(ii) Find the maximum value of $x + 2y$ subject to these conditions. [2]

9 You are given that $f(x) = x^3 - 4x^2 + x + 6$.

(i) Find the remainder when $f(x)$ is divided by $(x - 1)$. [1]

(ii) Show that $(x - 3)$ is a factor of $f(x)$. [2]

(iii) Hence solve the equation $f(x) = 0$. [4]

10 Find the coordinates of the points of intersection of the line $y = 5 - 2x$ with the curve $y = x^2 - 4x - 11$, giving your answers correct to 2 decimal places. [7]

[Turn over

Section B

11 It is known that 65% of all people living in the UK went abroad for a holiday last year.

A random sample of 5 people living in the UK was chosen.

Find the probability that

(i) all 5 went abroad for a holiday last year, [1]

(ii) exactly 4 went abroad for a holiday last year, [3]

(iii) at least 2 went abroad for a holiday last year. [4]

An additional random sample of 5 people living in the UK was chosen.

(iv) Find the probability that in the 10 people chosen altogether, exactly 8 went abroad for a holiday last year. [4]

12 A train normally travels between two points A and D at a constant speed of 30 metres per second. The distance AD is 12 kilometres.

(i) Find the time taken for the train to travel between A and D at $30\,\mathrm{m\,s^{-1}}$. [1]

Between A and D there are two other points, B and C, which are placed such that AB = 2 km, BC = 6 km and CD = 4 km. On one day there is a speed restriction of $10\,\mathrm{m\,s^{-1}}$ between B and C.

The train decelerates uniformly from $30\,\mathrm{m\,s^{-1}}$ at A to $10\,\mathrm{m\,s^{-1}}$ at B. It travels the distance BC at $10\,\mathrm{m\,s^{-1}}$. The train then accelerates uniformly from $10\,\mathrm{m\,s^{-1}}$ at C to $30\,\mathrm{m\,s^{-1}}$ at D.

Find

(ii) the time taken to travel from A to B, [2]

(iii) the acceleration over the distance CD, [3]

(iv) the extra time taken in travelling from A to D as a result of the speed restriction. [6]

13 Fig. 13.1 shows a solid block which is in the shape of a pyramid. The horizontal base, ABCD, is a square with side 20 cm and the vertex, V, is 15 cm vertically above the centre, O, of the square base. N is the midpoint of AB.

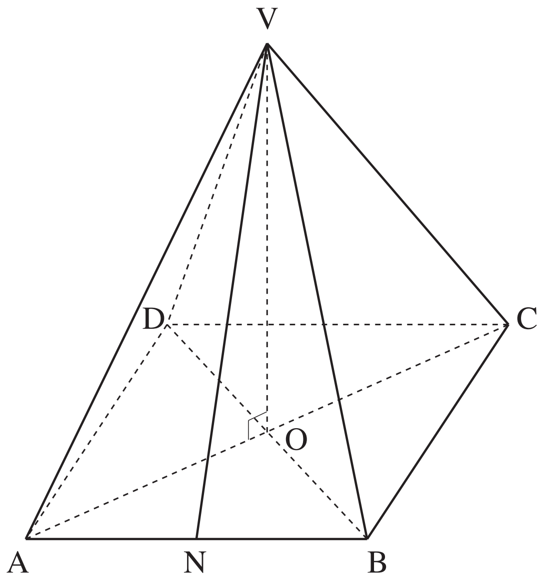

Fig. 13.1

(i) Calculate the length of the diagonal AC. [2]

(ii) Show that the length of the edge AV is $\sqrt{425}$ cm. [2]

(iii) Calculate the angle that the edge AV makes with the base. [2]

(iv) Calculate the length VN. [2]

M is the point on VB such that AM is perpendicular to VB as shown in Fig. 13.2.

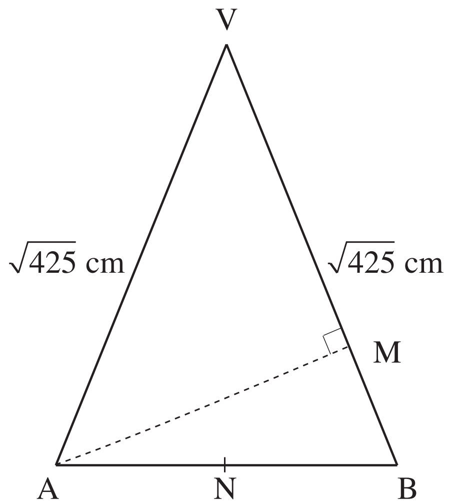

Fig 13.2

(v) Calculate the area of triangle VAB. Hence or otherwise calculate the distance AM. [4]

[Turn over

14

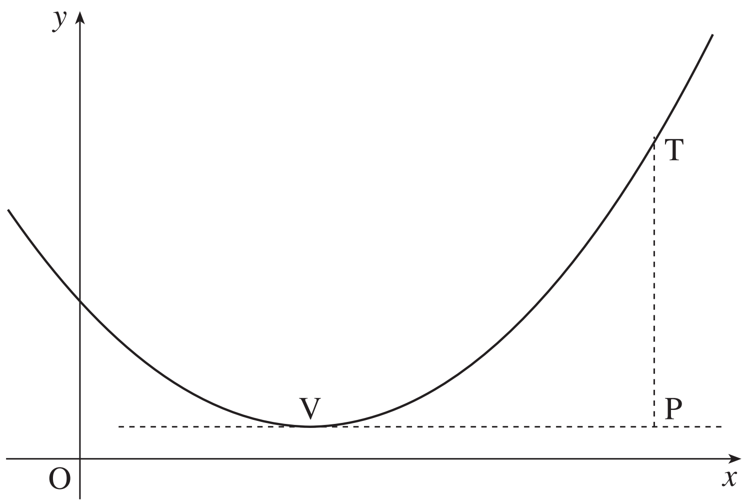

Fig. 14

Fig. 14 shows the quadratic curve $y = x^2 - 4x + 5$.

$V(2, 1)$ is the minimum point of the curve.

$T(5,10)$ is a point on the curve.

The line VP is the tangent to the curve at V and TP is perpendicular to this line.

(i) Write down the coordinates of P. [1]

(ii) Find the coordinates of M, the midpoint of VP. [2]

(iii) Find the equation of the tangent to the curve at T. [4]

(iv) Show that the tangent to the curve at T passes through the point M. [2]

(v) Use the result in part **(iv)** to suggest a way of drawing a tangent to a point on a quadratic curve without involving calculus. [3]

BLANK PAGE

BLANK PAGE