OCR
RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

6993/01

ADDITIONAL MATHEMATICS

FRIDAY 6 JUNE 2008

Afternoon

Time: 2 hours

Additional materials: Answer Booklet (16 pages)
Graph paper

You are not allowed a formulae booklet in this paper.

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 100.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages and **1** blank page.

[Turn over

Section A

1 A driver of a car, initially moving at $30\,\text{m}\,\text{s}^{-1}$, applies the brakes so that the car comes to rest with constant deceleration in 10 seconds.

(i) Find the value of the deceleration. [2]

(ii) Find the distance travelled in this time. [2]

2 The points A and B have coordinates (0, 8) and (6, 0) respectively.

(i) Find the equation of the line AB. [3]

(ii) Find the equation of the line perpendicular to AB through its midpoint. [4]

3 Find the points of intersection of the line $y = 5x + 13$ with the circle $x^2 + y^2 = 13$. [5]

4 Glass marbles are produced in two colours, red and green, in the proportion 7 : 3 respectively. From a large stock of the marbles, 5 are taken at random.

Find the probability that

(i) all 5 are red, [2]

(ii) exactly 3 are red. [3]

5 **(i)** Use calculus to find the stationary points on the curve $y = x^3 - 3x + 1$, identifying which is a maximum and which is a minimum. [6]

(ii) Sketch the curve. [1]

6 A speedboat accelerates from rest so that t seconds after starting its velocity, in $\text{m}\,\text{s}^{-1}$, is given by the formula $v = 0.36t^2 - 0.024t^3$.

(i) Find the acceleration at time t. [3]

(ii) Find the distance travelled in the first 10 seconds. [4]

7 A pyramid stands on a horizontal triangular base, ABC, as shown in Fig. 7.
The angles CAB and ABC are 50° and 60° respectively.
The vertex, V, is directly above C with VC = 10 m.
The angle which the edge VA makes with the vertical is 40°.

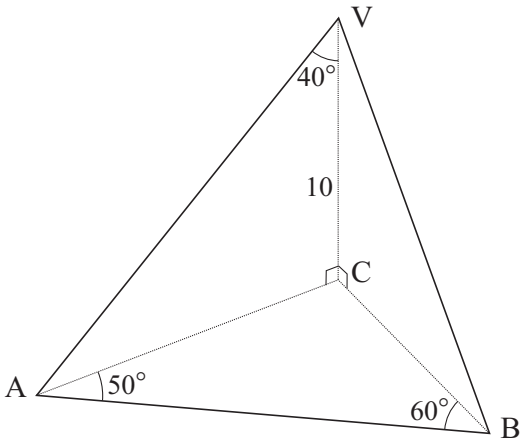

Fig. 7

(i) Calculate AC. [2]

(ii) Hence calculate AB. [4]

8 It is required to solve the equation $2\cos^2 x = 5\sin x - 1$.

(i) Show that this equation may be written as $2\sin^2 x + 5\sin x - 3 = 0$. [2]

(ii) Hence solve the equation $2\cos^2 x = 5\sin x - 1$ for values of x in the range $0° \leqslant x \leqslant 360°$. [4]

9 The cubic equation $x^3 + ax^2 + bx - 26 = 0$ has 3 positive, distinct, integer roots.

Find the values of a and b. [5]

[Turn over

Section B

10 Simon and Gavin drive a distance of 140 km along a motorway, both at constant speed. Simon drives at 5 km per hour faster than Gavin.

Let Gavin's speed be v km per hour.

(i) Write down expressions in terms of v for the times, in hours, taken by Gavin and Simon. [2]

Simon completes the journey in 15 minutes less than Gavin.

(ii) Explain why $\frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$ and show that this equation reduces to the equation

$$v^2 + 5v - 2800 = 0.$$ [5]

(iii) Solve this equation to find v and hence find the times taken by Simon and Gavin. Give your answers correct to the nearest minute. [5]

11 The side of a fairground slide is in the shaded shape as shown in Fig. 11. Units are metres.

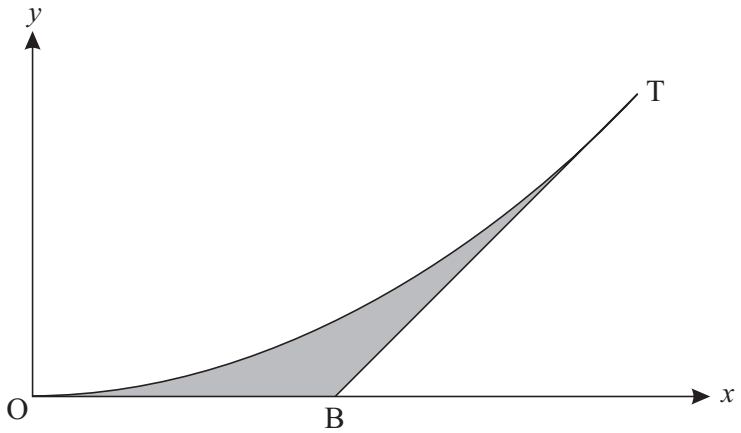

Fig. 11

The curve has equation $y = \lambda x^2$.

T has coordinates (4, 2). The line BT is a tangent to the curve at T. It meets the x-axis at the point B.

(i) Find the value of λ. [1]

(ii) Find the equation of the tangent BT and hence find the coordinates of the point B. [6]

(iii) Find the area of the shaded portion of the graph. [5]

12 A furniture manufacturer produces tables and chairs.

In each week the following constraints apply.

- There are 24 workers, each working for 40 hours (i.e. there are 960 worker-hours available).
- There is a maximum of £1800 available for the purchase of materials.
- Each table requires £30 worth of materials and 12 worker-hours.
- Each chair requires £10 worth of materials and 6 worker-hours.
- It is necessary to make at least 3 times as many chairs as tables.

Let x be the number of tables produced each week and y be the number of chairs produced each week.

(i) Show that the worker-hour constraint reduces to the inequality $2x + y \leqslant 160$. [2]

(ii) Find the inequality relating to the cost of materials constraint and the inequality relating to the numbers of tables and chairs. [3]

(iii) Plot these three inequalities on a graph, using 1 cm to represent 10 tables on the x-axis and 1 cm to represent 10 chairs on the y-axis. Indicate the region for which these inequalities hold. You should shade the region which is **not** required. [4]

When finished, each table is sold for a profit of £20 and each chair is sold for a profit of £5.

(iv) The manufacturer wishes to maximise the profit. Explain why the objective function is given by $P = 20x + 5y$. [1]

(v) Find the number of tables and chairs that should be made in order to maximise the profit. [2]

[Question 13 is printed overleaf.]

13 In the triangle shown in Fig. 13, M is the midpoint of BC.

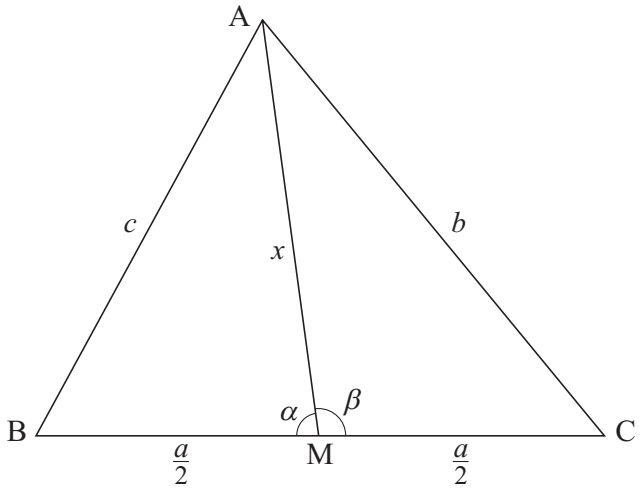

Fig. 13

(i) Explain why $\cos\alpha = -\cos\beta$. [2]

(ii) Using the cosine rule in the triangle BMA, show that

$$\cos\alpha = \frac{4x^2 + a^2 - 4c^2}{4ax}.$$ [2]

(iii) Find a similar expression for $\cos\beta$. [1]

(iv) Using the results in parts **(i)**, **(ii)** and **(iii)**, show that $4x^2 + a^2 = 2(c^2 + b^2)$. [5]

(v) A triangular lawn has sides 46 m, 29 m and 27 m. Find the distance from the midpoint of the longest side to the opposite corner. [2]

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