

FSMQ

Additional Mathematics

FSMQ **6993**

Mark Schemes for the Units

June 2009

6993/MS/R/09

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Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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Additional Mathematics – 6993

Section A

				_
1		Pythagoras for third value:	M1	Using any means to find $\sqrt{5}$
		$c = \sqrt{5}$	A1	
		$\Rightarrow \tan \theta = -\frac{\sqrt{5}}{2}$	A1 3	Includes negative sign.
		Alt:		
		$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{4}{9} = \frac{5}{9}$	M1	Use of Pythagoras
		$\Rightarrow \sin \theta = \frac{1}{3}\sqrt{5}$	A1	$\sin \theta$
		$\Rightarrow \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{5}/3}{\frac{2}{3}} = -\frac{\sqrt{5}}{2}$	A1	Includes negative sign
		SC: Allow B1 for $\tan \theta = -1.12$		
2		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 5$	M1	Attempt at differentiation with at least one term with
		\Rightarrow grad tangent = 8	A1	correct power
		\Rightarrow grad normal $=-\frac{1}{8}$	F1	
		$\Rightarrow y+1=-\frac{1}{8}(x-1)$	M1	Dep on use of their normal gradient and correct point
		$\Rightarrow 8y + 8 = -x + 1 \Rightarrow 8y + x + 7 = 0$	A1 5	Any acceptable form. Acceptable means three terms only
3	(i)	2x + 5y = 2 + 25	M1	Substitute new point to
		$\Rightarrow 2x + 5y = 27$	A1	change c
				If put in form $y = mx + c$
			2	then $m = -0.4$
		SC: B2 from scale drawing only if absolutely correct		
	(ii)	When $x = 3$, $6 + 5y = 27$	M1	Substituting $x = 3$ into either
	, ,	•		their equation from (i) or the given equation in (i)
		$\Rightarrow 5y = 21 \Rightarrow y = \frac{21}{5}$ $\Rightarrow p = \frac{21}{5} = 4.2$	F1 2	Answer must specifically give <i>p</i>
		NB $p = 0.2$ comes from using original line. Give M1 A1 for this.		

4	/i\	7, 2, 2			
4	(i)	$AB = \sqrt{(5-1)^2 + (3-1)^2}$	M1		
		$=\sqrt{4^2+2^2}$			
		$=\sqrt{20}=2\sqrt{5}$	A1	2	isw ie ignore any approx
		NB M1 A0 for 4.47 with no sight of $\sqrt{20}$		2	value for root.
	(ii)				
	(,	$\left(\frac{1+5}{2}, \frac{1+3}{2}\right) = (3,2)$	B1	1	
	(iii)	$(x \pm a)^2 + (y \pm b)^2$ with (a,b) from (ii)	M1		Use of equation
		$\left((x-a)^2 + (y-b)^2 \right)$	F1		Their midpoint
		= 5	A1		cao for 5
					isw ie ignore any incorrect algebra following a correct
				3	equation
	(1)	$v^2 = u^2 + 2as \Rightarrow 0 = 4 - 2 \times 0.25s$) / 1		Has of wish t for more 1 ()
5	(i)	$v = u + 2as \Rightarrow 0 = 4 - 2 \times 0.25s$	M1 A1		Use of right formula(e) Substitution
		0			
		$\Rightarrow s = 8$ Distance travelled = 8 m	A1	3	Answer
		If <i>t</i> is found first then M1 for any correct equations			
		that lead to finding s			
		Careful also of $4 = 0 + \frac{1}{2}s$, this could be 3 if			
		quoted formula is right.			
		Also of $0 = 4 + \frac{1}{2}s \Rightarrow s = -8$			
		Both of these M1 for formula only			
	(ii)	$s = ut + \frac{1}{2}at^2 = s = 3 \times 4 - \frac{1}{2} \times 0.25 \times 16$	M1		
		2 2	A1		
		= 12 - 2 = 10 Length of ramp = 10 m	A1		
			<u> </u> 	3	
6		NB Anything that uses $v = 0$ is M0			
0		$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 4x + 3x^2$	M1		For integrating - increase in power of one in at least two
		$\Rightarrow (y =) x - 2x^2 + x^3 (+c)$	A1		terms
		Through (2, 6)	M1		Attempt to find <i>c</i>
		$\Rightarrow 6 = 2 - 8 + 8 + c \Rightarrow c = 4$			
		$\Rightarrow y = x - 2x^2 + x^3 + 4$	A1	4	Must be an equation
				т	

		2 2 2	l	
7	(i)	$AC^2 = 8^2 + 3^2 - 2.8.3 \cos 60$	M1	Use of formula
		=73-24=49	A1	
		$\Rightarrow AC = 7$	A1	AC
		⇒ Total distance = 18 km	F1	Total distance
			4	
	(ii)	$\frac{\sin BCA}{\sin 60}$	M1	
	. ,	8 9		
		\Rightarrow sin BCA = $\frac{8}{9} \times$ sin 60 (= 0.7698)	A1	
		3 Shi Berr = 9 × Shi oo (= 0.7070)		
		\Rightarrow BCA = 50.3°	A1	
		Alternative Scheme:	3	
		Use of cosine formula twice	M1	
		\Rightarrow BC = 9.74	A1	
		$\Rightarrow BC = 9.74$ $\Rightarrow BCA = 50.3^{\circ}$	A1	
8		$2x + 11 = x^2 - x + 5$	M1	Substitute
"				
		$\Rightarrow x^2 - 3x - 6 \ (=0)$	A1	Quadratic
		$\Rightarrow x = \frac{3 \pm \sqrt{9 + 24}}{2} = \frac{3 \pm \sqrt{33}}{2}$	M1	Solve
		$\Rightarrow x = {2} = {2}$	A1	Correct substitution
		= 4.37 or -1.37	A1	Both answers
			5	Ignore values for <i>y</i>
		Alternative Scheme 1 (relates to last 3 marks)		
		Completion of square: $(x-1.5)^2 = k$	M1	
		$x - 1.5 = \pm \sqrt{8.25}$	A1	Must contain ±
		$\Rightarrow x = 4.37 \text{ or } -1.37$	A1	Must be 2 dp
		Alternative Scheme 2: Only 2 marks from last 3		
		Solving their quadratic by T&I	M1	
		Both roots	A1	
		Alternative Scheme 3. Only 4 marks		
		Roots with no working: B2 each	B2,2	
		<i>5</i>	,	
		Alternative Scheme 4. Only 4 marks		
		Finding a root from the original equations	M1	
		= one of them	A1	
		Finding the second root	M1	
		= the other	A1	
		Alternative scheme 5. Eliminate <i>x</i> .	M1	Eliminate <i>x</i>
		Gives $y^2 - 28y + 163 = 0$	A1	Quadratic
			M1	Solve
		Gives $y = 19.74$ and 8.26 leading to x values	A1	Both y values
			A1	Both x values
		NB Attempt to solve by graph - M0		

9	(i)	a = 4 - 0.2t	M1	Integrate (increase of power of one in at least one term)
				ŕ
		$\Rightarrow v = 4t - 0.1t^2$	A1	Ignore c
		$\Rightarrow v_5 = 20 - 2.5 = 17.5$		
		Velocity is 17.5 m s ⁻¹	A1 3	
	(ii)	At $t = 20$, $a = 0$	3	
		ie Maximum velocity	B1 1	
	(iii)	$v = 4t - 0.1t^2$	M1	Integrate their <i>v</i> from (i) (Increase in power of one term)
		$\Rightarrow s = \int_{0}^{20} 4t - 0.1t^{2} dt = \left[2t^{2} - 0.1 \frac{t^{3}}{3} \right]_{0}^{20}$	A1	Ignore c
		$= 2 \times 400 - 0.1 \times \frac{8000}{3} = 533.3 = 533$	A1 3	Allow exact answer or 3sf
		Distance travelled = 533 m		
10	(i)	10 7	B2,1	Lines, -1 each error
			B2,1	Shading, -1 each error Correct side of line. ft if gradient is the same sign.
	(ii)	y=2	4 E1	ft their graph
	(")	, 2	1	it then graph

Section B

11	(i)	$-x^2 + 8x - 9 = x^2 - 6x + 11$	M1	Equate
		$\Rightarrow 2x^2 - 14x + 20 = 0$	A1	Quadratic
		$\Rightarrow x^2 - 7x + 10 = 0$		
		$\Rightarrow (x-5)(x-2) = 0$	M1	Solve: Factorisation
		$\Rightarrow x = 2, 5$	A1	needs 2 numbers to multiply to their
		Substitute: $x = 2 \Rightarrow y = 4 - 12 + 11 = 3$		constant
		$x = 5 \Rightarrow y = 25 - 30 + 11 = 6$	A1 5	Or one pair, e.g.
		Alternative ochome		(2,3) or (5,6)
		Alternative scheme: Completion of square: $(x-3.5)^2 = k$	M1	
		$x-3.5 = \pm \sqrt{2.25}$		
		$x - 3.3 = \pm \sqrt{2.23}$ $\Rightarrow x = 5 \text{ or } 2$	A1	
		$\Rightarrow x = 5 \text{ or } 2$ $\Rightarrow y = 6 \text{ or } 3$	A1	
	(ii)	5 5	AI	
	()	$A = \int_{0}^{\pi} (y_1 - y_2) dx = \int_{0}^{\pi} (-2x^2 + 14x - 20) dx$	M1	Int between curves
		Γ ₂ ν ³ Γ ⁵	A1	± Correct
		$= \left -\frac{2x^3}{3} + 7x^2 - 20x \right ^3$	M1	expression
		L J2		Integrate their function (not if they
		$(2\times125, 7, 25, 100)$ $(16, 20, 40)$	A2	divide by 2)
		$= \left(-\frac{2 \times 125}{3} + 7 \times 25 - 100\right) - \left(-\frac{16}{3} + 28 - 40\right)$	112	
		$=\left(-\frac{250}{3}+75\right)-\left(-\frac{16}{3}-12\right)=-\frac{234}{3}+87=87-78=9$	M1	All terms, –1 for each error
		$\begin{pmatrix} 3 & 1 & 2 \\ 3 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 & 2 \\ 3 & 1 & 2 \end{pmatrix}$	A1 7	Sub into integral
		Alternative scheme:		Answer
		_	M1	Subtracting 2
		$A = \int_{2}^{5} (-x^{2} + 8x - 9) dx - \int_{2}^{5} (x^{2} - 6x + 11) dx$	M1	integrals Integrate either
		$\begin{bmatrix} x^3 \\ 1 \end{bmatrix}$	A1	
		$= \left[-\frac{x^3}{3} + 4x^2 - 9x \right]_2^5 - \left[\frac{x^3}{3} - 3x^2 + 11x \right]_2^5$	A1	All terms of y_1 All terms of y_2
		$= \left(\left(-\frac{125}{3} + 100 - 45 \right) - \left(-\frac{8}{3} + 16 - 18 \right) \right)$	M1	Substitute into
		$= \left(\left(-\frac{3}{3} + 100 - 43 \right) - \left(-\frac{3}{3} + 16 - 18 \right) \right)$		either integral
		$-\left(\left(\frac{125}{3}-75+55\right)-\left(\frac{8}{3}-12+22\right)\right)$	A1	
				For 18 or 9
		$=\left(13\frac{1}{3}-\left(-4\frac{2}{3}\right)\right)-\left(21\frac{2}{3}-12\frac{2}{3}\right)=18-9$	A1	
				Final answer
		$\mathbf{SC} \ A = \int (y_1 + y_2) dx M1 \text{ integrate and M1 sub only}$		
		$\int \int (y_1 + y_2) dx \text{with integrate and with sub-only}$		

		T			
12	(i)	$\frac{100}{BE} = \sin 30$	M1		Fraction right way
					up
		$\Rightarrow BE = \frac{100}{\sin 30} = 200 \text{ m}$	A 1		Correct expression
		$\Rightarrow BL = \frac{1}{\sin 30} = 200 \text{ m}$	A1		for BE
			AI	3	Or B3 if the special
					triangle is noticed.
		Alternative scheme:			
		$\frac{100}{BC} = \tan 30 \Rightarrow BC = \frac{100}{\tan 30} = 173.2$	M1		Ratio and
		$BC = \tan 30 \Rightarrow BC = \tan 30$	A1		Pythagoras
		$BE = \sqrt{100^2 + 173.2^2} = 200$	A 1		
		BE VIOC 11/3.2 200	111		Allow not exact
	(ii)	AE by Pythagoras:	M1		
		$AE = \sqrt{500^2 + 200^2} = 100\sqrt{29} = 538.5$	A1		soi
		$\sin A = \frac{100}{538.5}$	M1		
		$\Rightarrow A = 10.7^{\circ}$	A1		
		$\Rightarrow A = 10.7$	7 1 1	4	
		Alternative Scheme:			
		$BC = \sqrt{30000} \approx 173.2 \Rightarrow AC = \sqrt{280000} \approx 529.2$	M1		
		. 100	A1 M1		
		$\Rightarrow A = \tan^{-1} \frac{100}{\sqrt{280000}} = 10.7^{\circ}$	A1		
		NB $A = 10.9^{\circ}$ comes from $\sin^{-1} \frac{100}{\sqrt{280000}}$			
	(iii)	Area = $\frac{1}{2} \times 500 \times$ their BE	M1		
		_	A1		
		= 50 000	AI		
		Area = $\frac{1}{2} \times BG \times \text{ their AE}$	M1		
		2			
		\Rightarrow BG = $\frac{2 \times \text{their area}}{2 \times \text{their area}} = 185.7$ $\approx 186 \text{ m}$	A1 A1		
		their AE	AI	5	
		Alternative Scheme:	M1		
		Find angle A or E			
		Then $\frac{BG}{A} = \sin A \Rightarrow BG = 186 \text{ m}$	A1		
		500	A1		
		ie maximum 3 marks. The answer is found, but the			
		question says "Hence" and this is "otherwise".			
		NB If area is attempted but not used then give M1 A1.			
		If area is found after BG is found then do not mark it.			

		In all parts of this question allow answers to 3sf or 4 dp			
13	(a)	The selection is random.	B1		
	. ,	Allow anything that implies equal chance of selection		1	
	(b)(i)	P(all are female) = 0.6^6 (= 0.046656)	M1		Sight of 0.6 ⁶
		= 0.0467	A1		Must be 3 sf
				2	
	(ii)	$P(3 \text{ of each}) = Bin coeff \times 0.6^3 \times 0.4^3$	M1		One term with
					binomial coeff
		$= 20 \times 0.6^3 \times 0.4^3$	A1		20 (may be
			A1		implied)
					Powers (may be
		= 0.2765 or 0.276	A1		implied)
				4	
	(iii)	P(more females than males) = 6 , 0 or 5 ,1 or 4 ,2	M1		Add 3 terms
			B1		Binomial
					coefficients correct
					in at least two
		$= 0.6^6 + 6 \times 0.6^5 \times 0.4 + 15 \times 0.6^4 \times 0.4^2$	B1		terms
					Powers correct in at
		= 0.04666 + 0.1866 + 0.3110	B1		least two terms
					At least 2 terms
		= 0.5443	A1		correct.
		Allow 0.544, 0.545, 0.5444		5	
		Alternative scheme:			
		P(more females than males)	M1		Take 4 terms from
		= 1 - P(more males than females or equal numbers)	B1		1
		$= 1 - (0.4^{6} + 6 \times 0.4^{5} \times 0.6 + 15 \times 0.4^{4} \times 0.6^{2} + 20 \times 0.4^{3} \times 0.6^{3}$	B1		Binomial coeffs
		= 1 - (0.0041 + 0.0369 + 0.1382 + 0.2765)	B1		Powers
		= 0.5443	A 1		At least 2 terms
					correct
		The terms are:			
		0.0467, 0.1866, 0.3110, 0.2765, 0.1382, 0.0369, 0.0041			
		If P(more males than females), treat as MR and -2			
		If $p = 0.4$ and $q = 0.6$ then MR -2 (but also 0 for (b)(i)			
		where answer is given!)			

14	(a)(i)		B1	2	Line with +ve intercepts and –ve gradient Curve Condone +ve gradient for cubic at origin. Must pass through the origin
	(ii)	Can only intersect in one point.	B1	1	Allow if obviously true, even if one or both are wrong
		NB Do not allow if the curve implies that there could be more than one root but the line has not been drawn long enough - eg if curve is quadratic			
	(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 3$	B1		Correct two terms
		Greater than 0 for all x or attempt to solve their $\frac{dy}{dx} = 0$	M1		= 0
		so no solution to $3x^2 + 3 = 0$	A1	3	No solution
	(ii)	Because the curve is always increasing can only cross the <i>x</i> axis in one point which is the root	B1	1	There must be some reference to (b)(i)
	(c)(i)	By trial $f(2) = 0$ Condone $(x - 2)$ is a factor	B1	1	
	(ii)	$\Rightarrow (x-2)(x^2+2x+5)=0$	M1 A1	2	In long division at least x^2 must be seen
	(iii)	Discriminant " $b^2 - 4ac$ " = -16 < 0 So no roots. This means that $x = 2$ is the only root. NB "Quad does not factorise" is not good enough	B1	_1_	Depends on (ii) being correct
	(d)	The equation will only have one root (for all r and s .)	B1	1	Ignore extra comments even if wrong

Grade Thresholds

Additional Mathematics (6993) June 2009 Assessment Series

Unit Threshold Marks

Unit	Maximum Mark	Α	В	C	D	E	U
6993	100	73	63	53	44	35	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	В	С	D	E	U	Total Number of Candidates
6993	27.7	39.7	48.7	56.9	66.0	100	9859

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