Preparing for A-level Maths: Summer Task

Prerequisites

These topics are the key elements of the Higher GCSE Maths course that will be relied upon at A-level Maths.

Key Topics

1 Numbers and indices
- 1.1 Fractions
- 1.2 Surds
- 1.3 Indices

2 Algebra 1
- 2.1 Basic algebra
- 2.2 Solving linear equations
- 2.3 Linear inequalities
- 2.4 Forming expressions

3 Coordinate geometry 1
- 3.1 Straight-line graphs
- 3.2 The equation of a line
- 3.3 Mid-points and distances
- 3.4 Parallel and perpendicular lines
- 3.5 Intersections of lines

4 Algebra 2
- 4.1 Solving a quadratic equation by factorising
- 4.2 Completing the square
- 4.3 Solving a quadratic equation using the formula
- 4.4 Solving linear and non-linear equations simultaneously

5 Coordinate geometry 2
- 5.1 Transformations of graphs
- 5.2 Sketching curves
- 5.3 Intersection points of graphs
- 5.4 Three circle theorems

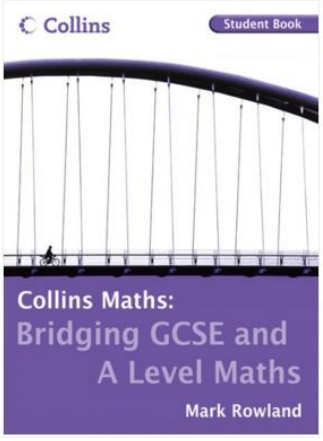
6 Trigonometry and triangles
- 6.1 Trigonometry and triangles
- 6.2 The area of any triangle
- 6.3 Solving a trigonometric equation

The questions that follow correspond to these sections.

They also correspond to the chapters of the textbook opposite, which is highly recommended for getting to grips with key topic areas from GCSE you will rely on for A-level.

The book provides detailed notes on each of these topics, full examples, questions, hints, common errors and full solutions.

Note: The book also includes two additional chapters on early A-level content (one on sequences, one on differentiation).



Collins Maths: Bridging GCSE and A Level Maths

CORE SECTION – A-level Maths Summer Task – CORE SECTION

Show your method, and unless otherwise stated, you should not use a calculator.

1 Numbers and indices

1.1 Evaluate:

$$\frac{1}{4} \div \frac{7}{8} + \frac{4}{9}$$

1.2a Simplify fully: $\sqrt{45} + \sqrt{180}$

1.3a Simplify fully: $(y^2)^3 \div y^4$

2 Algebra 1

2.1a Simplify as far as possible: $\frac{8x^6}{6x^8}$

2.1b Write as a single fraction in its simplest form: $\frac{2m}{n} - \frac{5m}{3n}$

2.2a Solve the equation: $\frac{4x}{5} - 3 = -21$

2.2b Solve the equation: $(x + 7) - 2(3 - 2x) = 16$

2.4 An isosceles triangle has two angles of $x°$. Find the size of the third angle in terms of x.

3 Coordinate geometry 1

3.1 Make a sketch of the two straight-line graphs: $y = 3 - 2x$ and $x + y = -1$

3.2 Find the gradient and the y-intercept of the line $2x + y = 4$.

3.3a Find the point exactly halfway between $(-2,5)$ and $(3,7)$.

3.3b Find the length of the line segment connecting $(-2,5)$ and $(3,7)$.

3.4a Write down the equation of a line parallel to $y = \frac{3}{2}x$.

4 Algebra 2

4.1a Solve the following equation: $x^2 - 5x - 24 = 0$

4.3 *[calculator permitted]* Solve the equation: $3x^2 = x + 5$

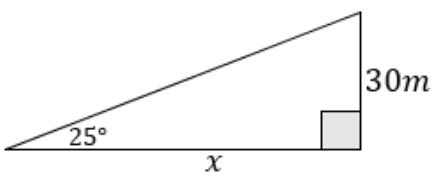
5 Coordinate geometry 2

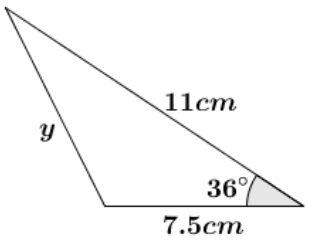
5.1a What transformation changes the curve $y = 3x^2$ into $y = 3x^2 + 5$?

5.2a Sketch the curve $y = x^2 - 9$, showing clearly where it crosses the coordinate axes.

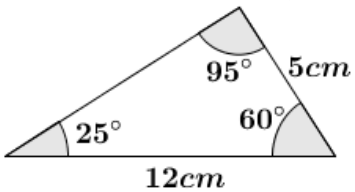
6 Trigonometry and triangles

6.1a *[calculator permitted]* Find the length of sides x and y in the triangles below:





6.2 *[calculator permitted]* Find the area of the triangle below:



EXTENSION SECTION – A-level Maths Summer Task – EXTENSION SECTION

Show your method, and unless otherwise stated, you should not use a calculator.

1 Numbers and indices

1.2b Write in the form $a + b\sqrt{3}$:

$$\frac{5}{1-\sqrt{3}}$$

1.3b Write in index form:

$$\frac{\sqrt{x}+1}{x^3}$$

2 Algebra 1

2.1c Make p the subject of the formula:

$$r(3-p) = \frac{p}{2}$$

2.3 Find the range of values for x which satisfies:

$$\frac{4-7x}{2} \leq -5$$

3 Coordinate geometry 1

3.4b A line perpendicular to $y = 3 - 2x$ goes through $(0,4)$. Find its equation.

3.5 Find the point at which the line $x + y = 5$ intersects the line $2y - x = 1$.

4 Algebra 2

4.1b Solve the following equation: $4x^2 - 4x = 15$

4.2a Write $x^2 - 8x + 200$ in the form $(x + p)^2 + q$ where p and q are integers.

4.2b Write down the minimum value that the expression $x^2 - 8x + 200$ can take.

4.4 Find any points where the line $x + y = 2$ crosses the curve $y = x^2 - 15x - 30$.

5 Coordinate geometry 2

5.1b What transformation changes the curve $y = x^3 - 3x^2$ into $y = 2x^3 - 6x^2$?

5.1c The original curve had minimum $(2,4)$. Where is the minimum of the new curve?

5.2b Sketch the curve $y = 3x - 2x^2$, showing clearly where it crosses the coordinate axes.

5.3 Where does the circle $x^2 + y^2 = 4$ cross the line $x + y = 2$?

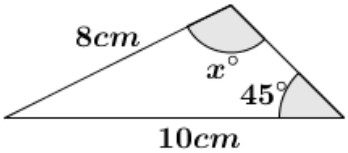
5.4a A circle crosses the x-axis at $(3,0)$ and $(a, 0)$, with centre $(5,1)$. Find a.

5.4b Find the radius of the circle, leaving your answer in exact form.

6 Trigonometry and triangles

6.1b A right-angled triangle has sides $4cm$ and $10cm$. What could the third side be?

6.1c *[calculator permitted]* Find the size of the obtuse angle labelled x in the triangle below:



6.3 *[calculator permitted]* Find all angles in the range $0° < x < 360°$ that satisfy $\sin x = \frac{1}{2}$.

Answer Sheet

*Write down **just the answers** in the space below, to facilitate marking and feedback.*
Your complete method and working should be kept separately so you can refer to it as needed.

Core Section	**Extension Section**
1.1	1.2b
1.2a	1.3b
1.3a	2.1c
2.1a	2.3
2.1b	3.4b
2.2a	3.5
2.2b	4.1b
2.4	4.2a
3.1 *(use squared paper on the next page)*	4.2b
3.2	4.4
3.3a	5.1b
3.3b	5.1c
3.4a	5.2b *(use squared paper on the next page)*
4.1a	5.3
4.3	5.4a
5.1a	5.4b
5.2a *(use squared paper on the next page)*	6.1b
6.1a	6.1c
6.2	6.3

Squared Paper

For use with graph sketching questions 3.1, 5.2a and 5.2b

Feedback

This is to be completed once your work is marked.

Core section *(Each question is worth 2 marks except 5.2a which is worth 3)*	**Extension section** *(Each question is worth 2 marks except 5.2a which is worth 3)*	**Grade** Core 50%: **Pass** Core 70%: **Merit** Core 70% & Ext 50%: **Distinction**
___ out of 20: ___ %	___ out of 20: ___ %	**Pass / Merit / Distinction**

HINTS CORE – A-level Maths Summer Task– CORE *HINTS*

1 Numbers and indices

1.1 Evaluate: $\frac{1}{4} \div \frac{7}{8} + \frac{4}{9}$

To divide by a fraction, multiply by its reciprocal. To add, first find a common denominator.

1.2a Simplify fully: $\sqrt{45} + \sqrt{180}$

To simplify a surd, find a square factor and use $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

To add things together, they must be of the same time (like $3x + 2x$ or $\frac{3}{7} + \frac{2}{7}$).

1.3a Simplify fully: $(y^2)^3 \div y^4$

To combine terms with powers, use the fact that $x^a \times x^b = x^{a+b}$.

Division is the opposite, and powers of powers can be treated as repeated multiplication.

2 Algebra 1

2.1a Simplify as far as possible: $\frac{8x^6}{6x^8}$

Number terms can be simplified by dividing top and bottom by any common factors.

Algebraic terms can be simplified in the same way, taking into account laws of indices.

2.1b Write as a single fraction in its simplest form: $\frac{2m}{n} - \frac{5m}{3n}$

To add or subtract fractions, first make the denominators the same.

2.2a Solve the equation: $\frac{4x}{5} - 3 = -21$

Consider the operations that are applied to x, and the order in which they apply.

Reverse the process, doing the opposite of each operation in the opposite order.

2.2b Solve the equation: $(x + 7) - 2(3 - 2x) = 16$

First multiply out any brackets (taking care with negatives) and collect like terms.

Next, isolate x terms on one side and numbers on the other.

2.4 An isosceles triangle has two angles of $x°$. Find the size of the third angle in terms of x.

Use the fact that angles in a triangle add up to $180°$ to form an expression for the third one.

3 Coordinate geometry 1

3.1 Make a sketch of the two straight-line graphs: $y = 3 - 2x$ and $x + y = -1$

Either identify x- and y-axis crossing points by setting $y = 0$ or $x = 0$, or rearrange to the form $y = mx + c$ to identify the gradient and y-intercept.

3.2 Find the gradient and the y-intercept of the line $2x + y = 4$.

Rearrange to the form $y = mx + c$ and use the fact that m is gradient, c the y-intercept.

3.3a Find the point exactly halfway between $(-2,5)$ and $(3,7)$.

The midpoint is the average in both the x and y directions, so take the mean of the x coordinates for the midpoint x coordinate, and the same thing for the y coordinate.

3.3b Find the length of the line segment connecting $(-2,5)$ and $(3,7)$.

Draw a sketch and identify the right-angled triangle whose hypotenuse is the line segment. Use Pythagoras to find the length of the segment.

3.4a Write down the equation of a line parallel to $y = \frac{3}{2}x$.

Parallel lines have the same gradient as one another. The y-intercept can be anything.

4 Algebra 2

4.1a Solve the following equation: $x^2 - 5x - 24 = 0$

Factorise into two brackets, then set each one separately equal to zero for two solutions.

4.3 *[calculator permitted]* Solve the equation: $3x^2 = x + 5$

Rearrange to give a 0 on one side, then use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

5 Coordinate geometry 2

5.1a What transformation changes the curve $y = 3x^2$ into $y = 3x^2 + 5$?

When the entire function has had something added, the result is a vertical translation.

5.2a Sketch the curve $y = x^2 - 9$, showing clearly where it crosses the coordinate axes.

Set $y = 0$ and solve for x to find x-axis crossing points, and set $x = 0$ to find y-axis crossing points. Finally, consider the type of function to determine the overall shape of the curve.

6 Trigonometry and triangles

6.1a *[calculator permitted]* Find the length of sides x and y in the triangles below *[see previous page for diagrams]*:

Use right-angled trigonometry ($\sin\theta = \frac{opp}{hyp}$, $\cos\theta = \frac{adj}{hyp}$, $\tan\theta = \frac{opp}{adj}$) for the first triangle.

Use cosine rule ($a^2 = b^2 + c^2 - 2bc\cos A$) since we know two sides and the included angle.

6.2 *[calculator permitted]* Find the area of the triangle below *[see previous page for diagrams]*:

Use the area formula $Area = \frac{1}{2}ab\sin C$ since we have two sides and the angle in between.

HINTS EXTENSION – A-level Maths Summer Task – EXTENSION HINTS

1 Numbers and indices

1.2b Write in the form $a + b\sqrt{3}$: $\frac{5}{1-\sqrt{3}}$

To rationalise the denominator, use the difference of two squares result: $(a-b)(a+b) = a^2 - b^2$.

1.3b Write in index form: $\frac{\sqrt{x}+1}{x^3}$

Split up the fraction (the reverse of adding fractions), then, as well as normal index rules, $\sqrt[n]{x} = x^{\frac{1}{n}}$.

2 Algebra 1

2.1c Make p the subject of the formula: $r(3-p) = \frac{p}{2}$

First deal with any brackets or fractions, then group p terms together, and take out p as a common factor.

2.3 Find the range of values for x which satisfies: $\frac{4-7x}{2} \leq -5$

Treat just like an equation, doing the same thing to both sides, other than the fact that you must avoid multiplying or dividing by a negative unless you also change $\leq$ to $\geq$.

3 Coordinate geometry 1

3.4b A line perpendicular to $y = 3 - 2x$ goes through $(0,4)$. Find its equation.

Perpendicular lines have opposite gradients, both in size and sign, so $m_2 = -\frac{1}{m_1}$.

3.5 Find the point at which the line $x + y = 5$ intersects the line $2y - x = 1$.

Lines intersect when both equations are true for a given value of x and y, so solve the equations simultaneously, either by the elimination method or by direct substitution.

4 Algebra 2

4.1b Solve the following equation: $4x^2 - 4x = 15$

Put all terms on the same side, then use the formula, completing the square or factorising to solve.

4.2a Write $x^2 - 8x + 200$ in the form $(x+p)^2 + q$ where p and q are integers.

Called 'completing the square'. Use the fact that $(x+p)^2 = x^2 + 2p + p^2$ to find p, then choose a suitable value of q to ensure the final expression is identical to the original one.

4.2b Write down the minimum value that the expression $x^2 - 8x + 200$ can take.

Since the smallest anything squared can be is 0, set this part of the completed square form to zero.

4.4 Find any points where the line $x + y = 2$ crosses the curve $y = x^2 - 15x - 30$.

Use direct substitution to replace y in the quadratic equation with an expression only involving x and numbers, then solve the resulting quadratic. Then sub back into $x + y = 2$ to find corresponding y values.

5 Coordinate geometry 2

5.1b What transformation changes the curve $y = x^3 - 3x^2$ into $y = 2x^3 - 6x^2$?

When the entire function has been scaled up by multiplication, it is the result of a vertical stretch.

5.1c The original curve had minimum $(2,4)$. Where is the minimum of the new curve?

Consider what happens to individual points during the course of a vertical stretch.

5.2b Sketch the curve $y = 3x - 2x^2$, showing clearly where it crosses the coordinate axes.

Set $y = 0$ and solve for x to find x-axis crossing points, and set $x = 0$ to find y-axis crossing points. Finally, consider the type of function to determine the overall shape of the curve.

5.3 Where does the circle $x^2 + y^2 = 4$ cross the line $x + y = 2$?

Use direct substitution from the linear equation into the circle equation and solve the resulting quadratic. Substitute your values back into the linear equation at the end to find corresponding coordinates.

5.4a A circle crosses the x-axis at $(3,0)$ and $(a,0)$, with centre $(5,1)$. Find a.

Draw a sketch and consider the symmetrical properties of circles.

5.4b Find the radius of the circle, leaving your answer in exact form.

Use Pythagoras to determine the distance from the centre to a point on the circle.

6 Trigonometry and triangles

6.1b A right-angled triangle has sides $4cm$ and $10cm$. What could the third side be?

Consider the two possibilities: $10cm$ is the length of the hypotenuse, or it isn't. Use Pythagoras' theorem.

6.1c *[calculator permitted]* Find the size of the obtuse angle labelled x in the triangle below*[see previous page for diagrams]*: Use sine rule ($\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$) since we know a side and the angle opposite it.

Note that there are sometimes two valid answers which add up to $180°$. Calculators give the acute one.

6.3 *[calculator permitted]* Find all angles in the range $0° < x < 360°$ that satisfy $\sin x = \frac{1}{2}$.

Use your calculator to find one solution, then sketch the graph of $y = \sin x$ and use its mirror symmetry.

Wider 'reading'

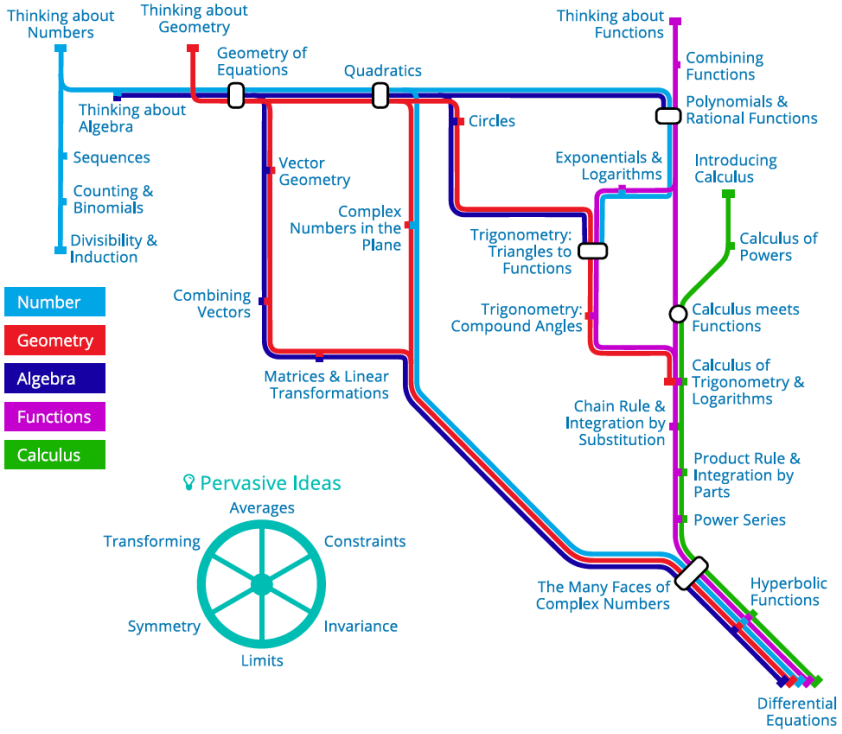
The mathematics you have come across so far only just scratches the surface of the subject.
As you study your A-level course you should discover and understand a whole lot more.

However, there is a huge amount of maths that you won't see in the classroom, and loads of interesting applications of the maths you *will* study that there simply won't be time to go into during lessons. There are lots of great books on different areas of mathematics, but there are also many excellent websites, videos and online resources you can get stuck into.

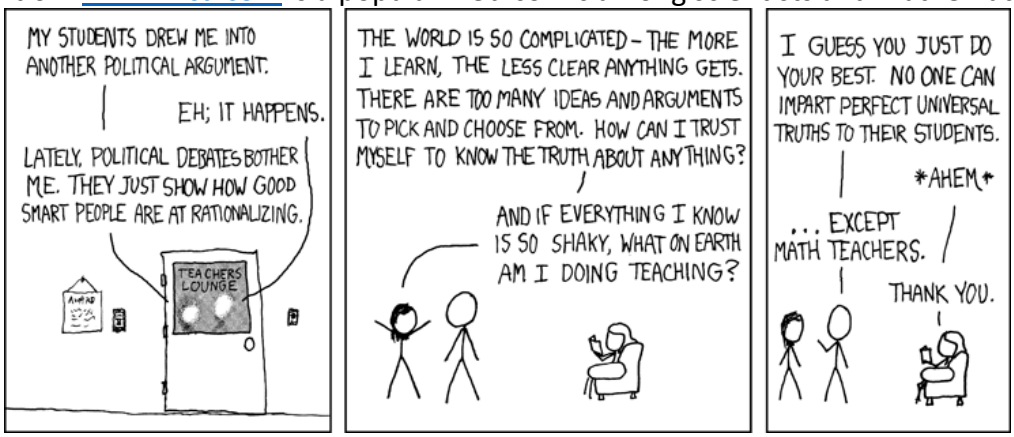
If you need a bit of inspiration, here are a few suggestions to get you started:

YouTube Channels	Maths Tools	Maths Problems
Numberphile	*GeoGebra.org*	*nrich.maths.org*
(interesting but accessible maths)	*(interactive geometry & graphs)*	*(broad problem solving tasks)*
ViHart	*WolframAlpha.com*	*Brilliant.org*
(quirky mathematician/musician)	*(mathematical search engine)*	*(short maths & logic problems)*
StandUpMaths	*codecademy.com*	*UKMT-resources.org.uk*
(stand-up comedian/mathematician)	*(free courses and coding platform)*	*(UK Maths Challenge questions)*

For more course-specific resources and rich tasks, this new initiative from Cambridge University, UndergroundMathematics.org, links the A-level topics, provides further details and encourages a deeper understanding through carefully crafted scenarios and problems:



Honourable mention: www.xkcd.com is a popular web-comic among scientists and mathematicians.



The what-if.xkcd.com section uses real science and maths to answer daft hypothetical questions.