OCR
RECOGNISING ACHIEVEMENT

FSMQ

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

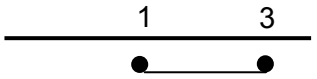
Viewing tips for this paper

In general, set your screen to ‘fit width.’

You may find it helpful to set to ‘fit height’ for the some questions:

[if you set a view, it stays for subsequent scripts]. If the writing is too small, you may wish to zoom in.

Section A

Question			Answer	Marks	Guidance	
1	(i)		$(x \pm 1)(x \pm 3)$ (≤ 0) $\Rightarrow 1 \leq x \leq 3$ **www**	**M1** **A1** **A1** **[3]**	Attempt to factorise **oe** Correct (or sight of 1 & 3) Answer	**SC** Test integers and select 1 and 3 **B**1 Accept $x \leq 3$ **and** $1 \leq x$ Or : from 1 to 3 **inclusive** (must imply **inclusion** of end points).
			Alternative: Draw curve for parabola the right way up **M1** Correct points on x-axis **A1** answer **A1**			
	(ii)			**B1** **[1]**	Correct answer Or: Follow through their ans to **(i)**.	Filled in circles must be evident. **SC B1** if correct but M0 in **(i)**. Accept alternative conventions. Answer must be a range (ie just a set of points is 0).

Question			Answer	Marks	Guidance	
2	(i)		$=1-\left(\frac{4}{5}\right)^5$ $=0.672(32)=\frac{2101}{3125}$	**M1** **A1** **[2]**	$1 - p^5$	p does not have to be 0.8 for this mark but the power must be 5. (ie p could be 0.2)
			Alternative: P(1) + ...+ P(5) 5 terms added, each term with powers correct **M1** Answer **A1**		Condone missing coeffs for **M1**	Terms are: 0.4096, 0.2048, 0.0512, 0.0064, 0.00032.
	(ii)		$10p^3q^2=10\times0.2^3\times0.8^2$ $=0.0512=\frac{32}{625}$ **www**	**M1** **B1** **A1** **A1** **[4]**	Must include powers of p and q and $\binom{5}{3}$ or ${}^5C_3$ (which need not be evaluated Powers Coefficient **soi** Accept 0.051 but not 0.05	Can be obtained by listing.

Question			Answer	Marks	Guidance	
3	(i)		f(3) = 12 $\Rightarrow$ 27+3a + 6 = 12 $\Rightarrow$3a = –21 $\Rightarrow$ a = –7	**M1** **A1** **[2]**		
			Alternative: Substitute a = –7 **M1** and show that R = 12 **A1**			If this method is used then if long division is used then $x^3 - 3x^2$ must be seen. NB Answer given so long division must be totally correct for A1
	(ii)		f(1) = 0 or (x – 1) seen $\Rightarrow$ f(x) = (x – 1)(x – 2)(x + 3)	**M1** **A1** **A1** **[3]**	Divide, try factor theorem for at least one value, or obtain a 3-term quadratic factor by inspection. Using or getting a correct factor or root Answer	Divide means you need to see the x^2 in the quotient and x^3 and x^2 terms correct in the initial dividing line.

Question			Answer	Marks	Guidance	
4			$s = \left(\frac{u+v}{2}\right)t \Rightarrow s = 13\times 10 = 130$ **www**	**M1** **A1**	In any order using any valid formulae. Ignore units	eg $s = \frac{(10+16)}{2}\times 10$
			$v = u + at \Rightarrow a = \frac{16-10}{10} = 0.6$ **www**	**M1** **A1** **[4]**		eg $16 = 10 + 10a$ Alternative order: $a = \frac{16-10}{10} = 0.6$ $\Rightarrow s = 10\times 10 + \frac{1}{2}\times 0.6\times 10^2 = 130$ **MR** u = 0 and v = 10 gives s = 50, a = 1 Or u= 0 and v= 16 gives s = 80 and a = 1.6 **M1 A0 M1 A0**

Question			Answer	Marks	Guidance	
5	(i)		$3-3\sin^2\theta = \sin\theta + 1$ $\Rightarrow 3\sin^2\theta + \sin\theta - 2 = 0$ **www**	**M1** **A1** **[2]**	Sight of and use of $\cos^2\theta = 1 - \sin^2\theta$ Must see = 0	NB answer given
	(ii)		$(3\sin\theta - 2)(\sin\theta + 1) = 0$ $\Rightarrow \sin\theta = -1$ or $\sin\theta = \frac{2}{3}$ $\Rightarrow \theta = 270°,\ 41.8°,\ 138.2°$	**M1** **A1** **A2** **[4]**	Solve to obtain $\sin\theta = \pm 1$ or $\sin\theta = \pm\frac{2}{3}$ Sight of both values All 3 with no extras in range Ignore $-90°$ **A1** for one or two values Or: all 3 values correct but extra values in range. Anything that rounds to 41.8° and 138° Allow 138° but not 42°	**SC2** $\sin\theta = 1, -\frac{2}{3}$ $\Rightarrow \theta = 90°, 318.2°, 221.8°$ (only) (Allow 318° and 222°)

Question			Answer	Marks	Guidance	
6	(i)		$\frac{dy}{dx} = 6x^2 - 18x + 12$ When $x = 2, \frac{dy}{dx} = 24 - 36 + 12 = 0$	**M1** **A1** **M1 dep** **A1** **[4]**	Differentiation All three terms Sub x = 2 into or factorise their derived function. Get 0 or set = 0 and get 2.	At least 2 terms with powers reduced by 1 (NB: beware division by x). **Do not condone division by 6 before substituting for x.** **NB answer given. Numerical values must be seen** Second M1 dep on first M1
	(ii)		$\frac{d^2y}{dx^2} = 12x - 18$ When $x = 2, \frac{d^2y}{dx^2} > 0$ giving a minimum	**M1** **A1** **[2]**	Diffn their derived function correctly. **BOD** no arithmetic computations seen.	Using the function $2x - 3$ can earn M1 A0.
			Alternative: Sign of gradient either side of x = 2 **M1** **Or:** Values of y either side of x = 2 **and** the value of y at stationary point. **M1** Correct answer (provided l.h. $x > 1$) **A1**		**BOD** no arithmetic computations seen. For **A1** LH x greater than 1	**Allow** sketch of function indicating left stationary value is maximum and right one is minimum**.**

Question			Answer	Mark	Guidance	
7	(i)		$(CB^2 =)\ 8^2 + 9^2 - 2\times 8\times 9\times \cos 20$ $= 9.684$ $\Rightarrow CB = 3.11$	**M1** **A1** **A1** **[3]**	8, 9 must be used, any angle **soi** Anything that rounds to 3.11	Ignore units
	(ii)		$\frac{\sin ABC}{8} = \frac{\sin \textit{their}\ 20}{\textit{their}\ 3.11}$ $\Rightarrow \sin ABC = 0.879$ $\Rightarrow ABC = 61.55°$ $\Rightarrow$ Bearing = 152°	**M1** **A1ft** **A1** **A1ft** **[4]**	Correct application of sine rule Must be same angle as used in **(i)** and *their* CB Anything that rounds to 62° **www** Anything that rounds to 152°	90 + their ABC
			Alternative methods: Cosine Rule: $\cos ABC = \frac{9^2 + \textit{their}\ CB^2 - 8^2}{2\times 9\times \textit{their}\ CB}$ $= 0.4767$ **M1 A1ft** Then angle and bearing **A1 A1ft** **OR**: Perpendicular from C and use of sin twice **M1** $h = 8\sin \textit{their} 20 = 2.736$ **A1ft** $\text{Sin}ABC = \frac{2.736}{\textit{their} CB}$ Then angle and bearing **M1 A1ft** **Or:** Find other angle by sine rule **M1 A1** Angle ACB = 98.45 giving ABC = 61.55 **A1** Bearing = 180 – (98.45 – 70) = 152 **A1ft**		Correct application of cos rule Must be same angle as used in **(i)** and *their* CB	NB Question asks for ABC so if not found 3/4 Angle = 81.55 can earn M1 A0 A0 (for ABC) A1ft only

Question			Answer	Marks	Guidance	
8	(i)		$\int_0^2 \left(x^2+2x-3\right)\mathrm{d}x = \left[\frac{x^3}{3}+x^2-3x\right]_0^2$ $= \left(\frac{8}{3}+4-6\right)-(0)$ **oe** $= \frac{2}{3}$ **www**	**M1** **A1** **A1** **[3]**	Integrate All three terms Completion to $\frac{2}{3}$.	Test for integration is "are there at least two terms with the power increased by 1?" Care that the process is not just multiplying each term by *x*. Working must be seen as the answer is given. Ignore absence of "– 0".
	(ii)		Because the curve crosses the *x*-axis in the range	**B1** **[1]**	Because one bit is +ve and the other is –ve.	Any reference to *x*= -3 will be 0. If there is an additional statement give 0.
	(iii)		$\left[\frac{x^3}{3}+x^2-3x\right]_0^1$ or $\left[\frac{x^3}{3}+x^2-3x\right]_1^2$ $= \pm 1\frac{2}{3}$ or $2\frac{1}{3}$ $\Rightarrow$ Total area $= 1\frac{2}{3}+2\frac{1}{3}=4$	**M1** **A1** **A1** **[3]**	Calculation of their integral between 0 & 1 or 1& 2 One of the areas	

Question			Answer	Marks	Guidance	
9	(i)		$h = 7 - 5 \times \cos 0 = 2$	B1 [1]		
	(ii)		$h = 7 - (-5) = 12$	M1 A1 [2]	Set $\cos\theta = -1$	
	(iii)		$9 = 7 - 5\cos(480t)$ $\Rightarrow \cos(480t) = -0.4$ **oe** $\Rightarrow 480t = 113.578$ $\Rightarrow t = 0.2366$ $\Rightarrow$ time $= 0.2366$ mins $= 14$ sec	M1 A1 A1 A1 [4]	Substitute $h = 9$ **soi**	 Allow 114 leading to $t = 0.2375$

Section B

Question			Answer	Marks	Guidance	
10	(i)		$(4,6)$	B1 [1]		
	(ii)		Distance MC: $\sqrt{(4-7)^2+(6-2)^2}$ $=5$ Equation of circle: $(x-4)^2+(y-6)^2=5^2\,(=25)$	M1 A1 M1 A1 [4]	Attempt to find radius or diameter by pythagoras. **soi** Must include their M and their r^2 Can be expanded form.	Answer given with no working then bod **B2** (ie $r = 5$, $r^2 = 25$, $d = 10$, $d^2 = 100$)
			Alternative: Equation of circle on AC as diameter: $(x-1)(x-7)+(y-10)(y-2)=0$ $\Rightarrow x^2-8x+7+y^2-12y+20=0$ $\Rightarrow (x-4)^2+(y-6)^2=25$ **isw**			
	(iii)		B lies on circle as $(8-4)^2+(9-6)^2=16+9=25$	B1 [1]		Working must be convincing
	(iv)		gradient of AM $=\left(\frac{10-6}{1-4}\right)=\frac{4}{-3}$ gradient of BM $=\left(\frac{9-6}{8-4}\right)=\frac{3}{4}$ Since $\frac{4}{-3}\times\frac{3}{4}=-1$ the lines are perpendicular	B1 B1 B1 [3]	One gradient (need not be simplified) Second gradient (need not be simplified) **Demonstration** that $m_1 \times m_2 = -1$ is satisfied and all **working to derive gradients shown.**	Labelling does not need to be specific. **SC** Both gradients upside down or signs the wrong way round **B0 B0 B1**

Question			Answer	Marks	Guidance	
			Alternative: Use of Pythagoras **M1** $5, 5, \sqrt{50}$ seen and used **A1** Arithmetic correct and final statement **A1**		Attempt to find all three lengths	
	(v)		$\text{B to M} = \begin{pmatrix}-4\\-3\end{pmatrix} \Rightarrow \text{M to D} = \begin{pmatrix}-4\\-3\end{pmatrix}$ $\Rightarrow$ D is $(0,3)$	**M1** **A1, A1** **[3]**	Idea of **BM = MD** soi Each value	
			Alternative: Centre as midpoint: Idea **M1** $\left(\frac{8+x}{2}\right) = 4 \Rightarrow x = 0$ Each value **A1 A1** $\left(\frac{9+y}{2}\right) = 6 \Rightarrow y = 3$			
			Alternative: Equation BM is $y = \frac{3}{4}x + 3$ Sub in eqn for circle $\Rightarrow x^2 - 8x = 0$ $\Rightarrow x = 0$ Sub to give $y = 3$ Idea **M1** Each value **A1 A1**			

Question			Answer	Marks	Guidance	
11	(i)		$\frac{dy}{dx} = x$ At A gradient of tangent = -2 so gradient of normal $= \frac{1}{2}$. $\Rightarrow$ Eqn of AB is $y - 2 = \frac{1}{2}(x+2)$ $\Rightarrow 2y = x + 6$ **oe**	**M1** **A1** **A1ft** **M1dep** **A1** **[5]**	Differentiation Follow through their gradient of tangent. Using (–2, 2) and their **normal** gradient 3 terms only	If no differentiation then 0/5
	(ii)		line meets curve when $x^2 = x + 6$ $\Rightarrow x^2 - x - 6 = 0$ $\Rightarrow (x-3)(x+2) = 0$ $\Rightarrow$ At B $x = 3,\ y = \frac{9}{2}$	**M1** **A1** **A1** **[3]**	Equate *their* straight line to given curve. Quadratic	
	(iii)		Area between = Area under line – area under curve $= 16.25 - 5.833 = 10.4$ $= 10\frac{5}{12}$	**M1** **M1** **M1dep** **A1** **[4]**	Attempt to evaluate area under curve by integration **soi** Attempt to evaluate area under their straight line by trapezium or integration **soi** Subtracting areas, dep on **both** M marks Answer	Seen by power increased by 1. Care not to multiply by x *Ignore absence of limits for first 3 marks*

Question			Answer	Marks	Guidance	
12	(i)		Substitute: 75 = 900*a* + 30*b* 240 = 3600*a* + 60*b* Solve: $\Rightarrow a = \frac{1}{20}, b = 1 \Rightarrow d = \frac{1}{20}v^2 + v$	**B1** **B1** **M1** **A1** **A1** **[5]**	Allow unsimplified coefficients Solve *a* *b*	 ie equal coefficients **and** subtract or correct substitution. NB Answers given so algebra for first value found must be convincing.
	(ii)		$D = \left(\frac{4900}{20} + 70\right) - \left(\frac{4225}{20} + 65\right)$ $= 38.75$	**M1** **A1** **A1** **[3]**	Calculation at each value and subtraction attempted For either 315 or 276.25 **soi** Allow 38.8	 Or 33.75 or 5
	(iii)		Substitute: $50 = \frac{1}{20}v^2 + v$ or $v^2 + 20v - 1000 = 0$ $\Rightarrow v = \frac{-20 \pm \sqrt{400 + 4000}}{2}$ ≈ 23.2 mph	**M1** **A1** **M1** **A1** **[4]**	Substitute Quadratic (in any form) **isw** Solving their quadratic using **correct** formula or completion of square **M1 A1** or **B2** answer with no working	Correct application of completion of square is $(v + 10)^2 = k$ seen **SCM1** for trial and improvement with values between 20 and 25. **A1** ans correct to 3 sf **SCB2** If answer given with no quadratic. Final answer is anything that rounds to 23.2 *Ignore negative values*

Question			Answer	Marks	Guidance	
13	(i)		$(2+h)^3 = 8+3.4h+3.2h^2+h^3$ $= (8+)12h+6h^2+h^3$	**B1** **B1** **B1** **[3]**	For each coefficient or term that is correct Ignore incorrect identification of coefficients after expansion Mark final line	ie allow answer left in simplified expansion form.
	(ii)		$(2 + h)^3 - 2^3$ $(2 + h) - 2$ Gradient $= \frac{(2+h)^3-8}{2+h-2} = \frac{(2+h)^3-8}{h}$	**B1** **B1** **B1** **[3]**	Change in *y* Change in *x* Only award if you are satisfied that the algebra is correct	Accept description in words
	(iii)		$\frac{(2+h)^3-8}{h} = \frac{8+12h+6h^2+h^3-8}{h}$ $= \frac{12h+6h^2+h^3}{h} = 12+6h+h^2$	**M1** **A1** **[2]**	Or using their part **(i)**	
	(iv)		*Their* 12 in (iii)	**B1** **[1]**	Dependent on **(iii)** being a polynomial.	This answer must be consistent with **(iii)**
	(v)		$(2+h)^4 = 16+32h+24h^2+8h^3+h^4$ Gradient of chord = $32 + 24h + 8h^2 + h^3$ Giving 32**www**	**B1** **B1** **B1** **[3]**	Allow $16 + 32h +$ (higher orders of *h*) Allow $32 +$ (higher orders of *h*) Dependent on previous work	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553